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ABSTRACT

This guide is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12. It is inclusive also of the objectives tested by the Stanford Achievement Tests and the Alabama Basic Competency Tests. One characteristic of the curriculum guide is that it clearly states what students should learn in each grade level. These objectives are called student outcomes and appear in the table of contents. The majority of the material, however, consists of classroom activities that lead students to the desired outcomes. The curriculum guide specifies what to teach as well as ways to teach it. Each section of the guide contains a specific table of contents, which lists what students should learn; activities; original sources; and student outcomes. Activities are preceded by a description of the area of mathematics covered, the specific student outcomes that are addressed in the activities, and notes for the teacher, which convey information important for effectively carrying out the activities. Topics include numeration, number theory, fractions, decimals, ratio, proportion, percent, geometry, graphs, statistics, probability, time, measurement, and money. (KR)

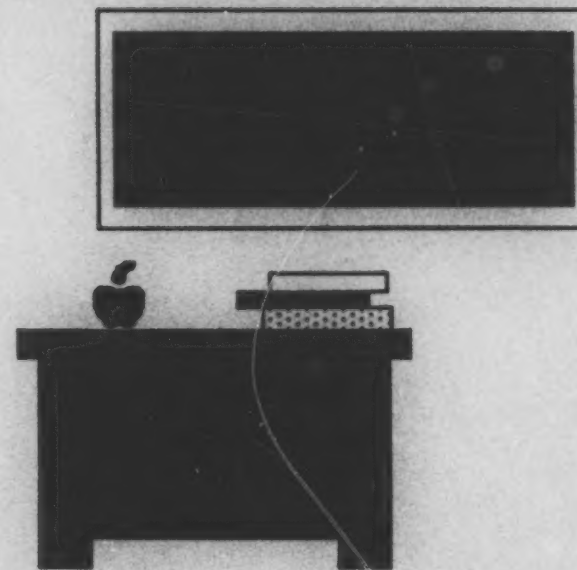
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MATHEMATICS

CURRICULUM GUIDE GRADES 7 and 8



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Bulletin 1987, No. 51

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MATHEMATICS CURRICULUM GUIDE CONSUMER MATHEMATICS

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FOREWORD

This Mathematics Curriculum Guide: Grades 7 and 8 is designed to help classroom teachers implement the Alabama Course of Study: Mathematics K-12 (Bulletin 1984, No. 16). It is inclusive also of the objectives tested by the Stanford Achievement Test and the Alabama Basic Competency Test.

One characteristic of the curriculum guide is that it clearly states what students should learn in 7th and 8th grade math. These are called STUDENT OUTCOMES and appear in the Table of Contents. The majority of the material, however, consists of classroom ACTIVITIES that lead students to the desired outcomes. The curriculum guide, then, specifies what to teach as well as ways to teach it.

The Mathematics Curriculum Guide: Grades 7 and 8 is supplemental in nature. It will add to material found in textbooks and complement locally developed skills continuums, scope and sequences, and/or curriculum guides. In the past, we have been most successful when we clearly defined what students should learn, gave examples of how it can be taught, and then tested for mastery. This curriculum guide is another step in that direction.

Wayne Teague
Wayne Teague

State Superintendent of Education

INTRODUCTION

The Mathematics Curriculum Guide: Grades 7 and 8 is one of a series of curriculum guides to be developed by the Alabama State Department of Education. Its production is part of a larger plan to provide LEAs with a curriculum guide in every subject area K-12, immediately following the Course of Study adoption in the same subject area. The Mathematics Curriculum Guide K-6 is packaged separately. Each of the other secondary mathematics guides (General Math, Consumer Math, Pre-Algebra, Geometry, Algebra I, Algebra II, Trigonometry/Analysis, and Applied Vocational Math) will be packaged separately also and will be published shortly.

The grades 7 and 8 mathematics guide has three distinct sections. It begins with a TABLE OF CONTENTS which lists what students should learn (student outcomes) in these two grades. After each outcome, a page number(s) directs the reader to classroom activities that address the specific outcome.

The second and largest section of the guide consists of classroom ACTIVITIES. Activities are preceded by a description of the area of 7th and 8th grade mathematics covered, the specific student outcomes that are addressed in the activities, and Notes for the Teacher that convey information important for effectively carrying out the activities.

The third section is referred to as the ORIGINAL SOURCES AND STUDENT OUTCOMES portion of the guide. It summarizes the relationship of student outcomes to the original sources from which the outcomes were taken. (The original sources for the Mathematics Curriculum Guide: Grades 7 and 8 outcomes were the Alabama Course of Study, the Stanford Achievement Test objectives, and the Alabama Basic Competency skills.) This section of the guide will serve as a reference for local school systems that may wish to trace the source of each outcome or to rate each outcome as being basic, standard, or advanced.

The use of the Mathematics Curriculum Guide: Grades 7 and 8 is not mandatory. Instead, it is offered as material that will help implement the Mathematics Course of Study; ensure curriculum validity for the Basic Competency Program; and prepare students for the Stanford Achievement Tests. The Mathematics Curriculum Guide: Grades 7 and 8 can be used as it is in LEAs that currently have no guide, or it can serve as a starting point for developing a similar guide locally. In either case, the involvement of local educators in the development of curricula is valued and encouraged. It is hoped that this guide will bring about local expansion activities such as writing outcomes for "optional" mathematics courses; adding to the list of outcomes for a given subject; writing additional activities in specific areas of identified weakness; rating outcomes as basic, standard, or advanced; and developing uniform end-of-unit tests.

ACKNOWLEDGEMENTS

From its inception, the development of curriculum guides has been viewed as a cooperative effort between specialists in the Division of Student Instructional Services and selected classroom teachers from the public schools of Alabama. The Curriculum Development Section, under the leadership of Katherine Mitchell, has the responsibility for coordinating this project.

Work on the Mathematics Curriculum Guide: Grades 7 and 8 started in January 1986, when Linda G. Harris was selected as Project Director and the identification of student outcomes began. The second phase of the project took place during July and August when five classroom teachers joined State Department of Education specialists in the writing of activities that teach the identified outcomes. For twelve days the following teachers offered their best ideas:

James Davidson	- Jasper City Schools
Patricia Edwards	- Etowah County Schools
Myrtle Johnson	- Eufaula City Schools
Ann Lilly	- Tuscaloosa City Schools
Wanda Motes	- Enterprise City Schools

Over a period of eleven months, a team of State Department of Education specialists identified and reviewed student outcomes, wrote and rewrote activities, read activities for clarity, and proofed various drafts. Robin Lanier contributed the graphics for the guide. Others who were directly involved with the Mathematics Curriculum Guide: Grades 7 and 8 were Kay Atchison, Delton Blalock, Jack Blucert, Moses Clark, Wanda Coleman, Robert Glass, Frank Heatherly, and Mary Jane Herring. Leigh Ann Lewis was the primary typist for the document; Rosemary Johnson assisted her on the first draft. We appreciate the work of all these people and their co-workers, who made the adjustments and sacrifices necessary to carry out their own programs and, at the same time, made the Mathematics Curriculum Guide: Grades 7 and 8 a division product.

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AREA: NUMBERS - NUMERATION - NUMBER THEORY

- OUTCOMES: 1. Read and write money values to \$10,000 and whole numbers to seven digits.
2. Read and write money values and whole numbers to nine digits.
3. Demonstrate an understanding of place value.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Have a basic knowledge of place values and decimals.

Steps to Success

1. The teacher should review place value.
2. A review of the concept of base ten numeration as it applies to our number system is helpful.

Begin by explaining that numbers may be divided into "families" of three digits each. These "families" are separated by commas and are called periods. Each "family" or period has a name. In any number (for example, 2,387,215) digits are divided from right to left into "families" or periods of 3 digits. Emphasize that a digit may be repeated in a number, and its position in the number determines the value.

A chart like the one below is helpful in showing the place value of each digit in the number.

TEN MILLIONS	MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
	2,	3	8	7,	2	1	5
MILLIONS FAMILY		THOUSANDS FAMILY			NO NAME FAMILY		

Steps to Success (Cont'd)

This chart may be extended to show larger numbers.

When digits have been correctly divided into "families" (periods), reading the number becomes easier. To divide a number into periods, count off each three-digit period starting with the last digit on the right and moving to the left. Count three digits and place a comma in front of the third digit. If there are more than three digits to the left of the comma, continue to group until no more than three digits are to the left of the comma.

ACTIVITY 1 - OUTCOMES 1, 2, and 3

Dictate or write on the board several word names for numbers. Use only the word names for two digits in the numbers ("three" and "four," for example). Vary the position of these digits to create the names of several numbers and ask students to write the corresponding numbers on paper at their desks. Have students check numbers they have written against the numbers written on the board. Examples could include:

- (a) three thousand four ----- 3,004
- (b) three hundred four ----- 304
- (c) thirty thousand, four ----- 30,004
- (d) thirty thousand, four hundred ----- 30,400
- (e) three million, three hundred, four ----- 3,000,304
- (f) three million, three hundred thousand, four ----- 3,300,004
- (g) four hundred three million, four hundred three thousand ----- 403,403,000
- (h) forty million, forty-three thousand, four hundred ----- 40,043,400

Notice that commas separate each "family" or period. Emphasize that zeros are used as place holders for place values for which digits are not named. When a digit is not named, one may assume that the "family member" is not home and a zero holds a place for this "family member" until he/she returns.

ACTIVITY 2 - OUTCOMES 1, 2, and 3

Reverse the process. Write a numeral (without commas) on the board.

Example 1: 3564281

- Solution:
1. Divide the number into periods or "families" of 3 digits. (3,564,281)
 2. Determine the highest place value family. (million)
 3. Read the number. (three million, five hundred sixty four thousand, two hundred eighty one)

Be sure to include examples with zeros.

Example 2: 2036300

- Solution:
1. Divide the number into "families." (2,036,300)
 2. Determine the highest place value family. (million)
 3. Read the number. (two million, thirty six thousand, three hundred)

Emphasize that zeros indicating missing or absent family members are not read.

ACTIVITY 3 - OUTCOMES 1 and 2

Remind students that the smallest coin in our monetary system is a penny. A penny is called "one cent" and has a value of .01 or 1/100 of a dollar. Our monetary system, therefore, only uses decimals to two places or to the hundredths place.

Dictate examples such as the following and have students write their corresponding money values.

Example 1: One thousand five hundred dollars and twenty cents
(Emphasize that the word "and" is an indication of the placement of a decimal point.)

Solution: \$1,500.20

Example 2: Five million, two hundred fifty-five dollars
Emphasize that the absence of the word "and" indicates that no cents are included in the example.

Solution: \$5,000,255
Emphasize that the three zeros indicate that the three members of the thousands "family" are not home.

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

OUTCOMES: 4. Correctly use signs of operations.
5. Correctly use order of operations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Know the signs of operations (+, -, x, ÷).
2. Know basic facts of addition, subtraction, multiplication, and division.
3. Understand the use of exponents.

Steps to Success

1. Review the basic signs of operations. Examples that use the same numbers but contain different operations are helpful in illustrating that the operation affects the answer.

$$15 + 5 = 20$$
$$15 \times 5 = 75$$

$$15 - 5 = 10$$
$$15 \div 5 = 3$$

2. Introduce the correct order of operations.

Step 1: Simplify all powers.

Step 2: Simplify all parentheses.

Step 3: Perform all multiplication and division in the order given (left to right).

Step 4: Perform all addition and subtraction in the order given (left to right).

Remind students that addition and subtraction may be performed before multiplication and division if and only if these operations are within parentheses.

Steps to Success (Cont'd)

3. Students must realize that mathematicians have agreed upon an order of operation in order to communicate through mathematical symbols. Students can best realize this fact through a series of examples (from very simple to more complex) that result in incorrect answers if the agreed-upon order of operations is not followed.

Example: $6 + 8 \div 2$

1. If the order of operations is followed and the division is performed first, then the correct answer is obtained (10).
2. If the order of operations is not followed, then communication does not take place, and an incorrect answer of 7 may be obtained.

ACTIVITY 1 - OUTCOMES 4 and 5

An important strategy is to design examples of increasing difficulty. Let students master the order of operations in simple problems, and then apply this knowledge in more complex problems.

USING FOUR OPERATIONS

Example 1: Evaluate $8 + 10 \div 2$.

Since the problem contains no powers or parentheses, the first step is to simplify the multiplication and division:

$$\begin{aligned} 8 + 10 \div 2 &= \\ 8 + 5 &= 13 \end{aligned}$$

The complexity of examples can be increased:

$$\begin{aligned} 8 + 10 \div 2 + 5 \times 2 &= \\ 8 + 5 + 10 &= 23 \end{aligned}$$

ACTIVITY 1 - OUTCOMES 4 and 5 (Cont'd)

ADDING PARENTHESES

Example 2: Evaluate $(8 + 10) \div 2$.

In this example, there are still no powers. The parentheses, however, change the problem. The first step is to simplify within the parenthesis.

$$18 \div 2 = 9$$

The complexity of examples can be increased:

$$\begin{aligned}(8 + 10) \div 2 + 5 + 2 \div 2 &= \\ 18 \div 2 + 5 + 1 &= \\ 9 + 5 + 1 &= 15\end{aligned}$$

ADDING POWERS

Example 3: Evaluate $5 + 4^2 + (8 + 4 \div 2) - 5 + 10 \div 2$.

$$\begin{aligned}5 + 4^2 + (8 + 4 \div 2) - 5 + 10 \div 2 &= \\ 5 + 16 + (8 + 4 \div 2) - 5 + 10 \div 2 &= \text{(Simplify powers)} \\ 5 + 16 + (10) - 5 + 10 \div 2 &= \text{(Simplify parentheses)} \\ 5 + 16 + 10 - 5 + 5 &= \text{(Divide since there is no multiplication)} \\ &= 31\end{aligned}$$

Remind students that when parentheses are being simplified, the same order of operations must be followed:

1) simplify powers; 2) simplify parentheses; 3) multiply and divide left to right; and 4) add and subtract left to right.

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

- OUTCOMES: 6. Translate word sentences to number sentences and vice versa.
7. Translate and solve number sentences.
8. Solve open sentences, equalities, and inequalities.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand the concept of variables.
2. Be proficient in addition, subtraction, multiplication, and division.
3. Be familiar with the equality symbol (=) and the inequality symbols listed below:

< less than	{ not less than
> greater than	} not greater than
≤ less than or equal to	≠ not equal to
≥ greater than or equal to	

Steps to Success

1. Start by showing students that what they hear or read (words) can be represented with mathematical symbols. For example, "six plus seven equals thirteen" can be written as $6 + 7 = 13$. Likewise, students need to be sure that they can "read" or "put into words" any number sentences that use the mathematical symbols listed above. Students will need to associate specific mathematical symbols with specific key words.

Operation

Key Words

+	sum, add, increased by, more, exceeded by, added to,...
-	difference, subtract, take away, fewer than, decreased by, diminished by,...
x	product, times, multiply, doubled, tripled,...
÷	quotient, divide, separate into...

Steps to Success (Cont'd)

2. Solving number sentences, equalities, and inequalities can be approached through using open sentences.

For example, Y increased by 4 equals 12 can be translated "something" increased by 4 equals 12.

It can be written $\square + 4 = 12$ (which is familiar to students) and then later written as $y + 4 = 12$.

ACTIVITY 1 - OUTCOME 6

Begin by having students translate number sentences into word sentences. Emphasize that one number sentence can be translated into several correct English statements.

For example, $8 + 4 = 12$ can be read:

1. Eight plus four equals twelve.
2. Eight plus four is twelve.
3. Eight increased by four equals twelve.
4. The sum of eight and four is twelve.
5. Eight added to four equals twelve.

Be sure to include subtraction examples. In such examples, the number to the left of the subtraction sign is not necessarily the first word name written.

For example, $12 - 8 = 4$ can be read:

1. Eight subtracted from twelve is four.
2. Eight fewer than twelve equals four.
3. Eight taken away from twelve leaves four.

Next, have students translate word sentences, read or heard, into written number sentences. Emphasize that not as many alternatives exist, yet there is usually more than one way to write the number sentence. For example, "Eighteen divided by six equals three" can be written: $18 \div 6 = 3$ or $18/6 = 3$.

ACTIVITY 1 - OUTCOME 6 (Cont'd)

"The product of six and two is twelve" can be written:

$$2 \times 6 = 12$$

$$2(6) = 12$$

$$6 \cdot 2 = 12$$

$$6 \times 2 = 12$$

$$6(2) = 12$$

$$2 \cdot 6 = 12$$

There is only one number sentence if a subtraction statement is given. For example: "six fewer than ten equals four" can be written:

$$10 - 6 = 4$$

ACTIVITY 2 - OUTCOME 7

In Activity 2 examples, one term of the number sentence is missing. Build on the students' ability to translate a word sentence into a number sentence. Extend this skill by including examples in which one term of the number sentence is missing. Use the "open sentence" representation first. At this point, have students solve number sentences by inspection.

WORD SENTENCE

"Something" increased by nine equals sixteen.

Seven plus nine equals "something."

The sum of seven and "something" equals sixteen.

OPEN NUMBER SENTENCE

$$\square + 9 = 16$$

$$7 + 9 = \square$$

$$7 + \square = 16$$

The example can be increased in difficulty

WORD SENTENCE

"Something" added to the product of six

and four equals 30.

Twice the sum of "something" and five equals 14.

OPEN NUMBER SENTENCE

$$\square + 6 \times 4 = 30$$

$$2(\square + 5) = 14$$

Move from using the open box to using a letter of the alphabet to translate a word sentence into an open number sentence. Provide examples that gradually increase in difficulty.

ACTIVITY 2 - OUTCOME 7 (Cont'd)

Translate and solve.

Word sentence: X increased by five equals 16.

Number sentence: $X + 5 = 16$

Solution: $X = 11$ (because $11 + 5 = 16$)

Translate and solve.

Word sentence: The product of six and y equals 24.

Number sentence: $6y = 24$

Solution: $y = 4$ (because $6 \times 4 = 24$)

Translate and solve.

Word sentence: Twice the difference between z and one equals fourteen.

Number sentence: $2(z - 1) = 14$

Solution: $z = 8$

Guide students in the following manner:

1. Two times "something" equals 14. (7)
2. What must be the value of the parenthesis? (7)
3. What number must replace z? (8)

ACTIVITY 3 - OUTCOME 6, 7, and 8

Introduce and/or review inequality signs. Then use the approach outlined above to

- (1) read and write number sentences involving inequalities, and
- (2) solve number sentences involving inequalities.

ACTIVITY 3 - OUTCOME 6, 7, and 8 (Cont'd)

Ask students to solve inequalities where the solution set is limited to whole numbers. Remind students that inequalities often have many solutions.

<u>Example</u>	<u>Solution</u>	
$x > 5$	6, 7, 8...	
$x \geq 5$	5, 6, 7...	
$y < 4$	3, 2, 1, 0	<u>NOTE:</u> Whole numbers are not negative.
$y \leq 4$	4, 3, 2, 1, 0	

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

OUTCOME: 9. Begin study of systems of numbers.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand and be able to graph whole numbers.
2. Understand and be able to graph natural (counting) numbers.
3. Understand fractions, integers, and their opposites.

Steps to Success

1. The lesson should begin with a thorough review of whole numbers and natural (counting) numbers.
2. A number line is helpful with this lesson. Show students where each type of number is located.
3. The definitions of rational and irrational numbers will be generated in the first two activities. It is important that students recognize these two terms as labels for subsets of the system of Real Numbers which have specific characteristics:
 - a. Rational numbers - All fractional numbers and their opposites
 - b. Irrational numbers - A number that cannot be expressed as a fraction

ACTIVITY 1 - OUTCOME 9

Introduce rational numbers by asking students "leading" questions similar to the following.

1. What are whole numbers? Give examples. List a set of whole numbers.
2. What are natural numbers? Give examples. List a set of natural numbers. What is the only number that is a whole number but not a natural (counting) number?
3. How can whole numbers be plotted on a number line?
4. Are there other points on the number line?
5. What do these other points represent?

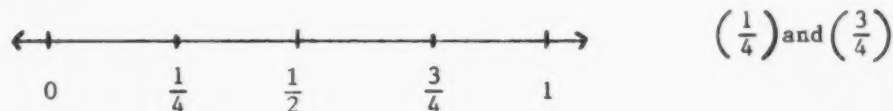
Begin with an enlarged number line.



What point is halfway between 0 and 1?



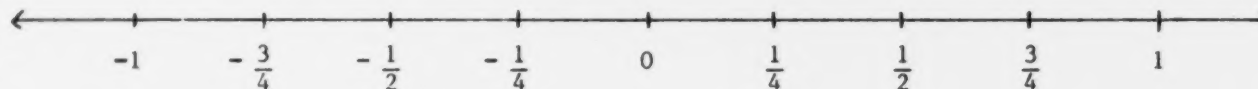
What point is halfway between 0 and $\frac{1}{2}$? between $\frac{1}{2}$ and 1?



ACTIVITY 1 - OUTCOME 9 (Cont'd)

Point out that this process could continue an infinite number of times. These new points $\left(\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\right)$ are fractional numbers and are part of the set called rational numbers.

Expand the number line to include integers and the opposites of the fractional numbers.



Introduce the definition of rational numbers as all numbers that can be expressed as fractions. Point out that different types of numbers make up the set of rational numbers because they can be expressed as fractions.

Examples: Whole numbers -- $2 = \frac{2}{1}$
Mixed numbers -- $2 \frac{1}{2} = \frac{5}{2}$
Square roots which are perfect squares -- $\sqrt{16} = 4 = \frac{4}{1}$
Percents -- $60\% = \frac{60}{100} = \frac{3}{5}$
Integers -- $6 = \frac{6}{1}$ and $-3 = -\frac{3}{1}$

ACTIVITY 2 - OUTCOME 9

Introduce irrational numbers.

Begin by referring to the rational number, $\sqrt{16}$, which is rational because it can be written as a fraction $\frac{4}{1}$.

Put $\sqrt{15}$ on the board and ask students to give the value of this number. Illustrate the fact that there is not a number which used as a factor two times will equal $\sqrt{15}$. The $\sqrt{15} = 3.873...$ The decimal is non-terminating and non-repeating.

ACTIVITY 2 - OUTCOME 9 (Cont'd)

This will then lead to the definitions of irrational numbers. An irrational number is a number that is both a non-terminating and non-repeating decimal, and it cannot be expressed as a fraction.

ACTIVITY 3 - OUTCOME 9

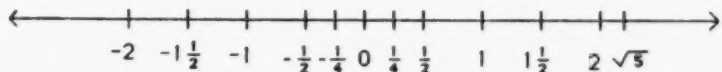
Ask students to classify these numbers as rational or irrational.

Examples: $\sqrt{18}$, 9, 20%, 75%, 0.25, $\sqrt{25}$, $\frac{8}{2}$

Many other examples may be given. Be sure to include numbers that are not usually written as fractions but may be expressed as fractions.

ACTIVITY 4 - OUTCOME 9

Draw a number line.



Develop the idea that all points on this line may be classified either as a rational number or as an irrational number. Therefore, the number line is a graph of a system of numbers called REAL NUMBERS. Real numbers are all the rational and all the irrational numbers. Emphasize that in the system of real numbers, a number is either rational or irrational. A number cannot be both rational and irrational.

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

OUTCOME: 10. Construct and use a number line to demonstrate signed numbers—integers.

NOTES FOR THE TEACHER

The activities for this outcome are found also in GENERAL MATHEMATICS - INTEGERS - OUTCOMES 1-8.

Prerequisites for Students

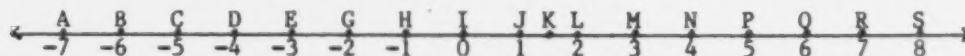
1. Identify whole numbers.
2. Read signed numbers.
3. Use the signs of inequality.

Steps to Success

1. The students' ability to understand that there are as many numbers to the left of zero as there are to the right of zero is a key to interpreting integers on a number line.
2. Ask students to note that as numbers move to the right on a number line, their values increase and as numbers move to the left on a number line, their values decrease.

ACTIVITY 1 - OUTCOME 10

Draw a number line with points labeled with letters of the alphabet.



Ask questions about the number line such as:

1. What point corresponds to -2? (G)
2. What point is halfway between J and P? (M or 3)
3. Label each inequality as true or false:
D < I (true)
-3 < -1 (true)
8 > -6 (true)

In the examples above emphasize that:

D < I means that D is to the left of I
-3 < -1 means that -3 is to the left of -1
8 > -6 means that 8 is to the right of -6

4. Which symbols (=, <, >) are needed to make the following statements true?

-5 _____ 0

E _____ -3

0 _____ A

5. What is the approximate value of K? (1 1/2)

ACTIVITY 2 - OUTCOME 10

Draw a number line on the board marking the line in equal increments. Locate zero, positive and negative numbers.

Associate the number line with examples from everyday life such as:

1. temperature on a thermometer;
2. profit and lost; or
3. loss and gain of yardage in football.

ACTIVITY 3 - OUTCOME 10

Define absolute value as the number of units a number is from zero on the number line.

On a number line, compare the distance of +4 and -4. Include examples to illustrate absolute value.

Show that $|4| = 4$ and $|-4| = 4$

$$|5| = 5 \qquad |7 + 2| = 9$$

$$|9 - 11| = 2 \qquad -|15 - 5| = -10$$

$$-2 + |-6| = 4 \qquad |10| - |7| = 3$$

Refer to the number line if needed.

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

OUTCOMES: 11. Use vocabulary and properties of integers to perform operations with whole numbers.

NOTES FOR THE TEACHER

Prerequisites for Students

The activities for this outcome are found also in GENERAL MATHEMATICS - INTEGERS - OUTCOMES 9-20.

1. Add, subtract, multiply, and divide whole numbers.
2. Have a knowledge of signed numbers.

Steps to Success

After completing the four activities described below, students should understand the following statements and apply these statements in computation problems involving integers.

1. When adding numbers with like signs, add the absolute values of the numbers and give the sum the common sign.
2. When adding numbers with unlike signs, subtract the absolute values of the numbers and give the difference the sign of the larger number.

NOTES FOR THE TEACHER (Cont'd)

3. When subtracting signed numbers, change the sign of the subtrahend and add.
4. When multiplying numbers with like signs, multiply the absolute values of the numbers and give the product a positive sign.
5. When multiplying numbers with unlike signs, multiply the absolute values of the numbers and give the product a negative sign.
6. When dividing numbers with like signs, divide the absolute values of the numbers and give the quotient a positive sign.
7. When dividing numbers with unlike signs, divide the absolute values of the numbers and give the quotient a negative sign.

The activities below describe situations which help students "see" why the statements above are true.

ACTIVITY 1 - OUTCOME 11

Draw a ladder in a hole.

Ask students to think of the step of the ladder located at ground level as zero.

All steps above ground level are positive.

All steps below ground level are negative.

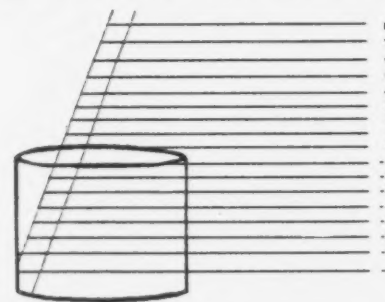
Ask student to represent examples such as the following on the ladder:

$$\begin{array}{ll} (+1) + (+3) = & (+3) + (+7) = \\ (-7) + (-2) = & (-5) + (-3) = \end{array}$$

Continue to give examples until students understand that adding numbers with like signs requires that the absolute values of the numbers be added and the sum be given the common sign.

Give examples such as the following and have students represent them on the ladder.

$$\begin{array}{ll} (+8) + (-2) = & (-7) + (+5) = \\ (+5) + (-6) = & (-8) + (+9) = \end{array}$$



ACTIVITY 1 - OUTCOME 11 (Cont'd)

Continue to give examples until students understand that adding numbers with unlike signs requires that the absolute values of the numbers be added and the sum be given the sign of the larger number.

ACTIVITY 2 - OUTCOME 11

Ask students to work the following subtraction problem just as they were instructed in elementary school.

Subtract:

$$\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$$

Ask students what the answer is. (4)

Then, give students the problem below and instruct them to add the integers.

Add:

$$\begin{array}{r} 6 \\ -2 \\ \hline \end{array}$$

Ask students what the sum is. (+4)

Ask students to look at the two answers and to observe that they are the same. Then lead students to the understanding that subtracting a number is the same as adding its opposite. Provide examples for practice using every combination of positive and negative integers.

Example 1: Subtract: $(+8) - (+2)$

Solution: $(+8) - (+2) = (+8) + (-2) = +6$

Include examples such as the following in student practice.

$$(+5) - (-2) = \quad (-3) - (-6) =$$

$$(+6) - (+4) = \quad (-8) - (+9) =$$

ACTIVITY 3 - OUTCOME 11

Ask students to imagine that they are standing on a street corner as an emergency vehicle passes down the street. The word AMBULANCE is written across the side of the vehicle from left to right.

Reading is a positive motion
(left to right)

Word is written in positive form
(left to right)

positive
(+)

of
X

positive
(+)

The result of a positive motion on a positive form is a positive image of AMBULANCE.

or

$$(+)(+) = +$$

Imagine that another ambulance comes down the street. This time you notice that the front has the letters ƎCNAJUBMA written on it.

Reading is a positive motion
(left to right)

Word is written backwards or in a negative form
(right to left)

positive
(+)

of
X

negative
(-)

The result of a positive motion on a negative form is a negative image of ƎCNAJUBMA .

or

$$(+)(-) = -$$

Ask students to pretend further that they are in a car and that same ambulance comes up behind them. This time, the driver looks out of the rear view mirror and sees AMBULANCE written on the front of the vehicle. Why?

ACTIVITY 3 - OUTCOME 11 (Cont'd)

Reading through the rear view mirror
(negative refraction)

Word is written backwards or in a negative form
(right to left)

negative
(-)

of
X

negative
(-)

The result of a negative motion on a negative form is a positive image of AMBULANCE.

or

$$(-)(-) = +$$

Give examples of multiplication problems involving signed numbers.

Example: $(-8) \times (+4)$

Solution: $(-8) \times (+4) = -32$

When multiplying numbers with unlike signs, multiply the absolute values of the numbers and give the product a negative sign.

or

$$(-)(+) = -$$

Include examples similar to the following in student practice.

Multiply: $\begin{array}{r} +5 \\ +7 \\ \hline \end{array}$

Multiply: $\begin{array}{r} -20 \\ +2 \\ \hline \end{array}$

Multiply: $(-3)(-18)$

Multiply: $(-2)(+1)(-3) =$

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

- OUTCOMES: 12. Use exponents to indicate the square of a number. Calculate the product.
13. Use exponents and extend to finding the root of a number.
14. Extend exponential use to finding powers and roots of perfect squares and cubes.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know how to use the basic operations.

Steps to Success

1. Understanding terminology such as exponents, base, factor, index, square, square root, cube, cube root, perfect square, and perfect cube is essential to a mastery of this area. Students may have been introduced previously to most of these terms and their mathematical notations. Therefore, the activities take the form of a thorough review.
2. Finding powers is approached as another way of representing something that students already know how to do -- find products. Therefore, calculating 2^3 is presented as a simple multiplication $2 \times 2 \times 2$.
3. Finding roots is approached as a way of "undoing" a perfect square or perfect cube. Therefore, $\sqrt{25}$ is thought of as the $\square^2$ in the following problems:

$$\square^2 = 25$$

or

$$\square \times \square = 25, \text{ where the unknown is the same number.}$$

ACTIVITY 1 - OUTCOME 12

Indicate that students can think of many examples where a number is multiplied by itself. Give a few examples and ask students for others (3×3 , 5×5 , 100×100). Indicate that another way of describing these examples is to say that a number (3, 5, 100) has been used as a factor 2 times. Introduce exponential notation, and have students provide numerous examples for a table such as the following:

Table of Squares

<u>FACTORS MULTIPLIED BY THEMSELVES</u>	<u>EXPONENTIAL NOTATION</u>	<u>PRODUCT</u>
5×5	5^2	25
10×10	10^2	100
3×3	3^2	9
100×100	100^2	10,000
2×2	2^2	4
8×8	8^2	64
$y \times y$	y^2	y^2

Be sure that students know how to "read" the column of exponential notations. For example, 5 is read "five squared" or "five to the second power".

Expand this activity by asking students if a number could be used as a factor more than two times. Have students provide examples for a table such as the following:

<u>BASES USED AS FACTORS</u>	<u>EXPONENTIAL NOTATION</u>	<u>PRODUCT</u>
$2 \times 2 \times 2$	2^3	8
$10 \times 10 \times 10$	10^3	1000
$3 \times 3 \times 3$	3^3	27
$10 \times 10 \times 10 \times 10 \times 10$	10^5	100,000
$2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6	64
$y \times y \times y \times y$	y^4	y^4

ACTIVITY 1 - OUTCOME 12 (Cont'd)

Point out in the previous examples that the exponent indicates how many times the base is used as a factor. For example, 2 indicates that the base (2) has been used as a factor six (6) times.

ACTIVITY 2 - OUTCOMES 12, 13, and 14

Approach finding roots as a way of "undoing" a perfect square or a perfect cube. Problems such as $\sqrt{16}$ can be approached as follows:

$$\square \times \square = 16 \quad \text{or} \quad \square^2 = 16$$

What must go in the box?

Problems such as $\sqrt[3]{8}$ can be approached as follows:

$$\square \times \square \times \square = 8 \quad \text{or} \quad \square^3 = 8$$

What must go in the box?

In the problem above, the 3 is called an index number. Its function is to tell how many times the given root must be used as a factor to obtain the numbers under the radical sign. In problems where an index number is not written, the index number is understood to be 2 (square root). When the index number is 3, it is often read "cubed root".

ACTIVITY 3 - OUTCOMES 12, 13, and 14

Since doubling a number is fairly easy, the powers of 2 can be generated easily.

$$\begin{aligned} 2 &= 2^1 = 2 \\ 2 \times 2 &= 2^2 = 4 \\ 2 \times 2 \times 2 &= 2^3 = 8 \\ 2 \times 2 \times 2 \times 2 &= 2^4 = 16 \end{aligned}$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10} = 1,024$$

ACTIVITY 3 - OUTCOMES 12, 13, and 14 (Cont'd)

After sufficient practice and familiarity, students should be able to answer immediately problems such as:

$$2^6 = \quad \sqrt[10]{1024} =$$

$$\sqrt[3]{8} = \quad 2^5 =$$

ACTIVITY 4 - OUTCOMES 12, 13, and 14

Finding roots can be extended to show that every positive number has two square roots (one positive and the other negative). This can be illustrated as follows:

$$\sqrt{16} = \pm 4 \text{ because}$$

$$4 \times 4 = 16 \quad \text{and}$$

$$-4 \times -4 = 16$$

The positive square root is called the principal root. Consequently, some situations may require the answer to $\sqrt{16}$ to be +4; the answer to $-\sqrt{16}$ to be -4; and the answer to $\pm \sqrt{16}$ to be ± 4 . Be sure to check rules pertaining to this notation if students are participating in math tournaments.

MATHEMATICS
GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

OUTCOMES: 15. Write numbers in expanded notation and use powers of ten (10) to write numbers in scientific notation.
16. Use scientific notation for expressing extremely large and small numbers.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand place value.
2. Understand that the numeration system most frequently used is written in base 10.
3. Have mastered operations with decimals.
4. Know the exponential form of the powers of 10. (Example: $1000 = 10^3$)
5. Understand that negative exponents indicate fractional values.

Steps to Success

1. Students will benefit from a brief review of place value with emphasis on the meaning of the decimal point. The decimal point separates whole numbers that are to the left of the decimal point from fractional values that are to the right of the decimal point. All numerals to the right of the decimal point represent values less than one.
2. An understanding of the exponential form of the powers of 10 is essential to an understanding of these activities. Therefore, the first activity deals with this area.

Steps to Success (Cont'd)

3. Some explanation of scientific notation and its usefulness in mathematics is needed before beginning Activities 2 and 3. Specifically, students should know that:
 - a. Scientific notation is used to write extremely large or extremely small numbers in a form that is shorter and easier to work with.
 - b. Scientific notation consists of two parts: a numeral between 1 and 10 that is multiplied by a power of 10.
 - c. Students must realize fully that scientific notation simply changes the form of a number; the value of the number remains the same.

ACTIVITY 1 - OUTCOME 15

Begin with a review of the exponential form of the powers of 10. Be sure that students understand that powers of 10 can be expressed in any of the three forms shown below.

<u>MULTIPLICATION</u>	<u>EXPONENTIAL FORM</u>	<u>NUMERAL</u>
10×10	10^2	100
$10 \times 10 \times 10$	10^3	1,000
$10 \times 10 \times 10 \times 10 \times 10$	10^5	100,000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^9	1,000,000,000

Expand examples to show students that numerals containing a digit other than one (1) can still be expressed as powers of ten.

<u>MULTIPLICATION</u>	<u>EXPONENTIAL FORM</u>	<u>NUMERAL</u>
$2 \times 10 \times 10$	2×10^2	200
$4 \times 10 \times 10 \times 10$	4×10^3	4,000
$6 \times 10 \times 10 \times 10 \times 10 \times 10$	6×10^5	600,000

ACTIVITY 1 - OUTCOME 15 (Cont'd)

Expand examples further to show students that numerals containing several digits other than one (1) can be written as the sum of numbers expressed as powers of ten.

<u>SUM</u>	<u>EXPONENTIAL FORM</u>	<u>NUMERAL</u>
$4,000 + 200$	$4 \times 10^3 + 2 \times 10^2$	4,200
$300,000 + 2,000$	$3 \times 10^5 + 2 \times 10^3$	302,000
$1,000,000 + 200$	$1 \times 10^6 + 2 \times 10^2$	1,000,200

Explain to students that when they are asked to write numbers in expanded form using powers of 10, they are required to express the numbers in the forms shown in the second column above.

Provide practice with examples such as the following:

Example: Write 12,000 in expanded form using powers of 10.

Solution: $12,000 = 10,000 + 2,000$
 $12,000 = (1 \times 10^4) + (2 \times 10^3)$

ACTIVITY 2 - OUTCOME 15

Expand Activity 1 to include decimal numbers. It is essential that students be able to convert decimals to fractions and vice versa.

$$.12 = \frac{12}{100}$$

$$.00012 = \frac{12}{100,000}$$

ACTIVITY 2 - OUTCOME 15 (Cont'd)

With this in mind, students can express decimals as fractions using the exponential form in the denominator.

$$.12 = \frac{12}{100} = \frac{12}{10^2}$$

$$.00012 = \frac{12}{100,000} = \frac{12}{10^5}$$

Be certain that students understand the correlation between fractional values $\left(\frac{1}{10}\right)$ and negative exponents (10^{-1}).

Provide practice with examples like the following.

Example: Write .0025 in expanded form.

Solution: $.0025 = \quad +$

$$.0025 = 2 \times \frac{1}{1000} + 5 \times \frac{1}{10,000}$$

$$.0025 = 2 \times \frac{1}{10^3} + 5 \times \frac{1}{10^4}$$

$$.0025 = 2 \times 10^{-3} + 5 \times 10^{-4}$$

ACTIVITY 3 - OUTCOMES 15 and 16

Explain that expanded notation expresses a number in a "spread out" and often cumbersome manner. Explain to students that scientific notation may be used to write extremely large and small numbers in a form that is shorter and easier to work with. (Review Steps to Success, #3).

Explain to students that scientific notation consists of two factors: a numeral between 1 and 10 and a power of 10. Illustrate this in several examples like the following.

Example: Write 12,000,000 in scientific notation.

Begin work with examples by asking leading questions.

1. Where is the decimal point in 12,000,000? (after the last zero on the right)
2. Where must the decimal point be re-located in order to have a number between 1 and 10? (1.2)
3. By what must 1.2 be multiplied in order for 1.2 to have the same value as the original number, 12,000,000? ($10,000,000$)
4. Write 10,000,000 in exponential form. (10^7)
5. Now, write 12,000,000 in scientific notation.

Solution: $12,000,000 = 1.2 \times 10^7$

Students will often see at this point that there is a "shortcut". Take 12,000,000 and move the decimal from its original location to a location that will name a number between 1 and 10. 12,000,000.

This gives the first part of the scientific notation. 1.2

Then count the number of places the decimal point was moved. (7) This is the power of 10 by which the 1.2 must be multiplied in order that the value of the original number is not changed. Therefore, $12,000,000 = 1.2 \times 10^7$.

NOTE: Since the original number is a whole number, the exponent must be positive because only positive exponents give whole numbers.

ACTIVITY 4 - OUTCOMES 15 and 16

Extend scientific notation to decimal numbers. Provide practice with examples like the following:

Example: Write .000015 in scientific notation.

Again, begin work with examples by asking leading questions.

1. Where must the decimal point in .000015 be moved in order to have a number between 1 and 10? (1.5)
2. By what must 1.5 be multiplied in order for 1.5 to have the same value as the original number .000015? $\left(\frac{1}{100,000}\right)$
3. Write $\frac{1}{100,000}$ in exponential form. $\left(\frac{1}{10^5} \text{ or } 10^{-5}\right)$
4. Now write .000015 in scientific notation. $(.000015 = 1.5 \times 10^{-5})$

Students should see the shortcut in this example. Move the decimal point from its original location to a location that will name the number between 1 and 10. (.000015)

This gives the first part of scientific notation. (1.5)

Then count the number of places the decimal point was moved (5). This is the power of 10 by which 1.5 must be multiplied in order that the value of the original number is not changed. Since the original number is a fractional number, the exponent (power of 10) must be negative because only negative exponents give fractional numerals.

$$.000015 = 1.5 \times 10^{-5}.$$

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

- OUTCOMES: 1. Add and subtract whole numbers to seven digits with regrouping.
2. Multiply and divide with multipliers and divisors up to three digits.
3. Multiply and divide with zero and one in multipliers, divisors, and dividends.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Have mastered the basic number facts.

Step to Success

1. Remind students of the importance of correctly aligning the numbers in the problem. Students who continually have difficulty with this problem may benefit from practice drills using graph paper where the numbers are written with one digit per square.

ACTIVITY 1 - OUTCOME 1

Example: Add $2,365 + 213,586 + 9,234,567$

Solution: Write the numbers in columns, correctly aligning the ones digits, the tens digits, the hundreds digits, etc.

			1	1	2	1	
				2,	3	6	5
		2	1	3,	5	8	6
+	9,	2	3	4,	5	6	7
	9,	4	5	0,	5	1	8

ACTIVITY 2 - OUTCOME 1

Example: Subtract 527,502 from 2,356,987

Solution: Write the numbers in columns, correctly aligning the digits.

	1	1	4	1			
	~~2~~	3	~~5~~	6	9	8	7
-		5	2	7	5	0	2
	1	8	2	9	4	8	5

ACTIVITY 3 - OUTCOME 2

Example: Multiply 1,232 x 421

- Solution:
1. Write the smaller number as the multiplier.
 2. Begin to multiplying the ones digit of the multiplier times all digits in the multiplicand.
 3. Continue by multiplying the digits in the tens and hundreds place times all digits in the multiplicand. Indent one place to the left each time a new partial product is written.
 4. After all digits of the multiplier have been multiplied by the digits in the multiplicand, add these partial products to find the final answer.

		1	2	3	2	
	x		4	2	1	Step 1
		1	2	3	2	Step 2
	2	4	6	4		Step 3
4	9	2	8			Step 3
5	1	8	6	7	2	Step 4

ACTIVITY 4 - OUTCOME 3

Example: Multiply 2367×201

Solution: Follow the procedure in Activity 3.

		2	3	6	7	
	x		2	0	1	Step 1
		2	3	6	7	Step 2
	0	0	0	0		Step 3
4	7	3	4			Step 3
4	7	5	7	6	7	Step 4

Alternate Procedure: Time can be saved by eliminating the line of zeros. Point out that a zero occupies the tens place in the multiplier. Since any number multiplied by zero equals zero, it is unnecessary to show multiplication by zero. However, students must realize that the 2 in the multiplier is in the hundreds place; therefore, the first digit in the second partial product must be indented 2 places.

		2	3	6	7
	x		2	0	1
		2	3	6	7
4	7	3	4		
4	7	5	7	6	7

ACTIVITY 5 - OUTCOMES 2 and 3

Division of large numbers will be easier if the steps of division are followed. They are easily remembered if the steps are thought of as members of a family.

STEPS OF DIVISION

1. Divide - Dad
2. Multiply Down - Mom
3. Subtract - Sister
4. Bring Down - Brother

ACTIVITY 5 - OUTCOMES 2 and 3 (Cont'd)

Provide practice with examples containing one-digit divisors. In the first examples, make sure that students can follow the algorithm correctly.

Example: $2 \overline{)564}$

Solution:
$$\begin{array}{r} 2 \\ 2 \overline{)564} \\ \underline{4} \\ 16 \end{array}$$

Divide: 2 goes into 5 two times. Write 2 in the quotient's place above the 5.

Multiply Down: 2 times 2 equals 4.

Subtract: 5 minus 4 equals 1.

Bring Down: 6 is the next digit. Bring it down beside the 1 to make 16.

$$\begin{array}{r} 28 \\ 2 \overline{)564} \\ \underline{4} \\ 16 \end{array}$$

Divide: 2 goes into 16 eight times. Write the 8 in the quotient's place above the 6.

Multiply Down: 8 times 2 equals 16.

Subtract: 16 minus 16 equals 0.

Bring Down: 4 is the next digit. Bring it down beside the 0.
04 is the same as 4.

$$\begin{array}{r} 282 \\ 2 \overline{)564} \\ \underline{4} \\ 16 \\ \underline{16} \\ 04 \\ \underline{4} \\ 0 \end{array}$$

Divide: 2 goes into 4 two times. Write the 2 in the quotient's place above the 4.

Multiply Down: 2 times 2 equals 4.

Subtract: 4 minus 4 equals 0.

Bring Down: Since there are no digits remaining in the dividend to bring down, the division is completed.

The answer is 282.

ACTIVITY 5 - OUTCOMES 2 and 3 (Cont'd)

Expand practice to include examples with divisors of two or more digits and examples with remainders.

Example: Divide: $756 \div 35 =$

Solution:

$$\begin{array}{r} 2 \\ 35 \overline{) 756} \\ \underline{70} \\ 56 \end{array}$$

Divide: 35 will not go into 7, therefore; divide 35 into 75. 35 goes into 75 two times. Write the 2 in the quotient's place above the 5 in 75.

Multiply Down: 2 times 35 equals 70.

Subtract: 75 minus 70 equals 5.

Bring Down: 6 is the next digit. Bring it down beside the 5. Now there is 56.

$$\begin{array}{r} 21 \text{ R}21 \\ 35 \overline{) 756} \\ \underline{70} \\ 56 \\ \underline{35} \\ 21 \end{array}$$

Divide: 35 goes into 56 one time. Write the 1 in the quotient's place above the last 6 in the dividend.

Multiply Down: 1 times 35 equals 35.

Subtract: 56 minus 35 equals 21.

Bring Down: There are no digits remaining to bring down. Since the remainder (21) is less than the divisor (35), the division is completed.

The answer is 21 with a remainder of 21.

ACTIVITY 5 - OUTCOMES 2 and 3 (Cont'd)

Expand examples to include problems with zero in the dividend and in the quotient. Problems should also include divisors with three or more digits.

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

OUTCOME: 4. Give examples of the properties of addition and multiplication and order of operations.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Be familiar with natural numbers, even numbers, odd numbers, and whole numbers.

Steps to Success

1. Mathematical properties are laws that tell us what we can or cannot do.
2. Students should identify each property and correctly spell each property's name.

ACTIVITY 1 - OUTCOME 4

TYPE 1: PROPERTIES OF ADDITION

1. The Commutative Property of Addition states: When adding numbers, the addends may be reordered without changing the value of the expression. Ask students what the word "commute" means. Attention should be drawn to the "back and forth" idea.

The Commutative Property of Addition represented algebraically is $a + b = b + a$. Notice that addends can be reordered.

Example 1: $6 + 8 = 8 + 6$

Example 2:
$$\begin{array}{r} 7 \\ +2 \\ \hline \end{array} = \begin{array}{r} 2 \\ +7 \\ \hline \end{array}$$

ACTIVITY 1 - OUTCOME 4 (Cont'd)

2. The Associative Property of Addition states: When adding numbers, the addends may be regrouped without changing the value of the expression.

Ask students what is implied by the phrases: "Doctors Smith, Jones, and Associates," or "National Rifle Association," or "Alabama Education Association," or "I will not associate with you any more!"

Attention should be drawn to the words "group" or "together."

The Associative Property of Addition represented algebraically is $(a + b) + c = a + (b + c)$.

Notice that addends can be regrouped..

$$\begin{array}{rcl} (2 + 4) + 5 & = & 2 + (4 + 5) \\ 6 + 5 & = & 2 + 9 \\ 11 & = & 11 \end{array}$$

3. The Identity Property for Addition states: When adding zero, the sum is the identical original number.

Ask students what is implied by words such as: "identification", "identify", "identical twins."

Bring students' attention to the idea of "the same as."

The Identity Property for Addition represented algebraically is $a + 0 = a$.

Example: $3 + 0 = 3$ or $0 + 3 = 3$

Emphasize that the Identity Property guarantees that if 0 is added to 3, the answer will be 3. Therefore, the identity element for addition is 0.

ACTIVITY 1 - OUTCOME 4 (Cont'd)

TYPE 2: PROPERTIES OF MULTIPLICATION

1. Commutative Property of Multiplication

Ask students to state again what the word "commute" means, and refer them to the Commutative Property of Addition. Then ask students to state the Commutative Property for Multiplication. Definitions should emphasize that when multiplying, the factors may be reordered without changing the value of the expression.

The Commutative Property of Multiplication represented algebraically is $a \times b = b \times a$.

Example : $4 \times 5 = 5 \times 4$
 $20 = 20$

2. Associative Property of Multiplication

Ask students to state again what the words "associate" and "associates" mean, and refer them to the Associative Property of Addition. Then ask students to state the Associative Property of Multiplication. Definitions should emphasize that in multiplication, the factors can be regrouped.

The Associative Property of Multiplication represented algebraically is $(ab)c = a(bc)$.

Example: $(2 \times 5) \times 4 = 2 \times (5 \times 4)$
 $10 \times 4 = 2 \times 20$
 $40 = 40$

3. Identity Property for Multiplication

Ask students what number they can multiply any other number by to get the same number.

Example: $\square \times 5 = 5$

Solution: $1 \times 5 = 5$

Therefore, the identity element for multiplication is 1.

The Identity Property for Multiplication stated algebraically is $a \cdot 1 = a$ and $1 \cdot a = a$.

ACTIVITY 1 - OUTCOME 4 (Cont'd)

TYPE 2: PROPERTIES OF MULTIPLICATION (Cont'd)

4. Inverse Property for Multiplication

Ask students what they can multiply a number by to get 1.

We multiply a number by its reciprocal to get 1.

Examples: $2 \times \frac{1}{2} = 1$ The reciprocal of 2 is $\frac{1}{2}$.

$\frac{1}{3} \times \frac{3}{1} = 1$ The reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$.

Emphasize that any number multiplied by its reciprocal gives a product of 1.

The Inverse Property for Multiplication stated algebraically is $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.

TYPE 3: THE PROPERTY OF BOTH ADDITION AND MULTIPLICATION SIMULTANEOUSLY

The Distributive Property of Multiplication with Respect to Addition is a law that states: The product of a number (a) and the sum of two other numbers (b and c) is equal to the sum of the products of the first number multiplied by each of the other two numbers.

Example: $5 \times (2 + 6) = (5 \times 2) + (5 \times 6)$
 $5 \times 8 = 10 + 30$
 $40 = 40$

The Distributive Property of Multiplication with Respect to Addition stated algebraically is $a(b + c) = ab + ac$.

ACTIVITY 2 - OUTCOME 4

Ask students if they have ever met "My Dear Aunt Sally."

Write this expression on the board, capitalizing the first letter in each word.

Explain that the first letter in each word represents one of the four basic operations (multiply, divide, add, subtract). This particular order of operations tells which operation should be used first in evaluating an expression such as:

$$24 + 8 \div 2 - 7 \times 3$$

My Dear Aunt Sally tells that students should first multiply or divide as indicated in the problem from left to right, and then, add or subtract as indicated in the problem from left to right.

Example: $24 + 8 \div 2 - (7 \times 3)$
 $24 + (8 \div 2) - 21$
 $(24 + 4) - 21$
 $28 - 21$
 7

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

OUTCOME: 5. Use addition and division to find averages of whole and decimal numbers.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Master the skills needed to add and divide decimals and whole numbers.

Step to Success

1. Emphasize that there are 3 basic types of average problems. In some problems addition precedes division, but in other problems no addition is necessary.

ACTIVITY 1 - OUTCOME 5

The key to finding averages is recognizing the word average in the problem. Once the word average is seen, the student looks for one of the three types of problems for finding averages.

TYPE 1: A series of numbers appears in the problem and the average must be found. All numbers are added and the sum is divided by the number of addends.

Example 1: (Type 1) Larry scored 83, 78, 72, and 87 on four mathematics tests. What was his average score?

ACTIVITY 1 - OUTCOME 5 (Cont'd)

Solution: Add the four scores

83	(1)
78	(2)
72	(3)
87	(4)
320	

Divide the sum (320) by the number of addends (4).

80	
4	320
32	
0	
0	

ANSWER: 80

Type 2: Only two numbers may appear in this type of problem. This type of problem may be easier to solve because there are only two numbers, not a series of numbers.

Example 2: In Japan a new kind of train travels 217.5 miles in three hours. What is the average speed in miles per hour for the train?

Solution: The problem has given a total number of miles traveled (217.5 miles) and a total amount of time traveled (three hours). An average number of miles traveled in one hour must be determined.

$$\frac{217.5 \text{ miles}}{3 \text{ hours}} = \frac{? \text{ miles}}{1 \text{ hour}}$$

To find the number of miles traveled in one hour, simply divide the total number of hours traveled (3) into the total number of miles traveled (217.5).

72.5	
3	217.5
21	
7	
6	
15	
15	
0	

72.5 miles were traveled in 1 hour or the train traveled at 72.5 mph.

ACTIVITY 1 - OUTCOME 5 (Cont'd)

Type 3: Sometimes only one number will be given in a problem. Other needed information will come from facts that are already known.

Example 3: A man has a monthly salary of \$900.00. On the average, how much did he earn each week?

Solution: The problem has given a total number of dollars earned (\$900.00) and a total amount of time for which the money was earned (1 month). The amount of money earned for one week must be determined. To set up a proportion, the number of weeks in a month must be determined. Four weeks in a month is the fact that is not given and must already be known.

$$\frac{900 \text{ dollars}}{4 \text{ weeks}} = \frac{? \text{ dollars}}{1 \text{ week}}$$

To find the number of dollars earned in a week, simply divide the total number of weeks worked (4) into the total number of dollars earned (900).

$$\begin{array}{r} \$225.00 \\ 4 \overline{) \$900.00} \\ \underline{8} \\ 10 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

\$225.00 were earned in 1 week or
the man earned an average of \$225 per week.

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

- OUTCOMES: 6. Estimate sums, differences, products, and quotients to computational problems.
7. Solve problems using approximations to find sums, differences, products, and quotients.

AREA: FRACTIONS AND DECIMALS

- OUTCOMES: 10. Round decimals.
11. Estimate sums, differences, products, and quotients.

AREA: APPLICATIONS

- OUTCOME: 1. Solve word problems about realistic situations involving whole numbers, fractions, and/or decimals requiring the use of one or more operations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Add, subtract, multiply, and divide whole numbers and decimals.
2. Round decimals and whole numbers to a given place value.

Steps to Success

1. Problems involving money values provide good opportunities for students to estimate sums, differences, products, and quotients.
2. Problems involving money values have some practical significance, since students often have a certain amount of money and need to estimate whether or not they have sufficient funds to make a particular purchase.
3. Examples 1 and 2 require students to estimate sums and products. Other examples should be provided which require students to estimate differences and quotients.

ACTIVITY 1 - OPERATIONS WITH WHOLE NUMBERS - OUTCOMES 6 and 7
 FRACTIONS AND DECIMALS - OUTCOMES 10 and 11
 APPLICATIONS - OUTCOME 1

Example 1: Motor oil at C-Mart is on sale for \$.59 a quart. Four quarts will cost about

- (a) \$.60
- (b) \$.50
- (c) \$2.40
- (d) \$2.00

Solution: To make this estimate, round \$.59 to \$.60. It is easier to multiply 4 times \$.60 than to multiply 4 times \$.59. The answer is \$2.40 (c).

Example 2: Use the advertisement below. Round costs to the nearest ten cents. Find the total estimated cost for all items. Decide whether or not \$6.55 is sufficient for purchasing all items.

Ruled paper	\$.89
Clipboard	.97
Pack of pens	.64
Desk lamp	3.98

- Solution:
- 1) \$. 89 Round \$.89 to \$.90.
 - 2) \$. 97 Round \$.97 to \$1.00.
 - 3) \$.64 Round \$.64 to \$.60.
 - 4) \$3.98 Round \$3.98 to \$4.00.
 - 5) The addition is easier after numbers have been rounded.

$$\begin{array}{r}
 \$.90 \\
 1.00 \\
 .60 \\
 +4.00 \\
 \hline
 \$6.50
 \end{array}$$

is the total estimated cost.

- 6) All items could be purchased with \$6.55.

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

OUTCOME: 8. Select reasonable answers to problems using distance, capacity, and weight.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be familiar with measures of distances such as inches, feet, miles, and meters; measures of weights such as ounces, pounds, tons, and grams; and measures of capacity such as cups, liters, and pints.
2. Have mastered the basic operational skills in addition, subtraction, multiplication, and division of whole numbers and fractions.

Step to Success

1. Emphasize that an estimate is close to the actual answer.

ACTIVITY 1 - OUTCOME 8

In examples like the following, tell students to choose the answer that is a close estimate.

Example 1: 22 inches is about 2 _____.

- (a) meters
- (b) feet
- (c) liters
- (d) yards

Solution: There are 12 inches in one foot and 24 inches in two feet. The correct answer would be (b) because it is the closest estimate.

ACTIVITY 1 - OUTCOME 8 (Cont'd)

Example 2: A 17 ounce steak weighs about one _____.

- (a) milligram
- (b) gram
- (c) liter
- (d) pound

Solution: One pound is equal to 16 ounces. Since 17 ounces is just a little more than a pound, (d) is the closest estimate.

NOTE: The examples in Activity 1 are taken from the Basic Skills Instructional Materials: Mathematics, Bulletin 1984, No. 36.

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

- OUTCOMES: 9. Identify composite and prime numbers and write the prime factorization of numbers.
10. Determine least common multiple (LCM) and greatest common factor (GCF).

NOTES FOR THE TEACHER

Prerequisites for Students

1. Use the basic operations competently.
2. Identify the multiples of a number.
3. Identify the factors of a number.

Steps to Success

1. Be sure that students thoroughly understand prime and composite numbers before beginning to teach GCF and LCM.
2. Teach the students various methods of finding LCM and GCF. Students may then choose the method they will use.

ACTIVITY 1 - OUTCOME 9

Begin by introducing definitions of prime and composite numbers.

1. Prime number - a number that has only 2 factors - itself and one (1) . The number 1 is not prime.
2. Composite number - a number that has more than two different factors.

Be sure to emphasize that the number 1 is neither prime nor composite. It is not composite because it does not have 2 different factors. The only factor of one (1) is one (1).

Introduce the Sieve of Eratosthenes as a method of determining prime numbers. This method may be used with numbers as great as you choose. The following exercise will yield the prime numbers between 1 and 50.

ACTIVITY 1 - OUTCOME 9 (Cont'd)

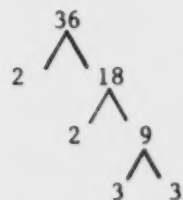
- Step 1: Make a chart listing the numbers between 1 and 50.
- Step 2: Cross out 1 because it is not prime.
- Step 3: Circle 2 which is prime (factors = 1 and 2). Cross out all other multiples of 2. They are composite because their factors are 1, 2, and the number itself.
- Step 4: Circle 3, which is prime, but cross out all other multiples of 3. Be sure that students can explain why 3 is circled and the multiples of 3 are crossed out.
- Step 5: Circle 5, which is prime, but cross out all other multiples of 5. Have students explain why some multiples of 5 are already crossed out.
- Step 6: Circle 7, which is prime, but cross out all other multiples of 7.
- Step 7: Circle all other numbers. Have students prove that they are prime.

~~1~~	(2)	(3)	~~4~~	(5)	~~6~~	(7)	~~8~~	~~9~~	~~10~~
(11)	~~12~~	(13)	~~14~~	~~15~~	~~16~~	(17)	~~18~~	(19)	~~20~~
~~21~~	~~22~~	(23)	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	(29)	~~30~~
(31)	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	(37)	~~38~~	~~39~~	~~40~~
(41)	~~42~~	(43)	~~44~~	~~45~~	~~46~~	(47)	~~48~~	~~49~~	~~50~~

NOTE: If you choose to expand this number chart, continue to circle the next prime number (11) and cross out all multiples of 11. Then circle 13 and cross out all multiples of 13, etc.

ACTIVITY 2 - OUTCOME 9

Use factor trees as a method of determining prime factorization. Demonstrate the procedure using an example such as 36.



Always begin with the lowest prime factor (2 in this case).

Use the lowest prime factor again if possible.

When the lowest prime factor (2) is no longer a factor, go to the next lowest prime factor (3).

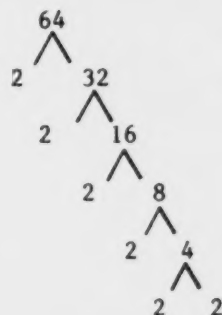
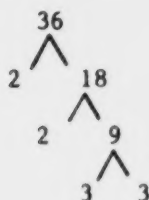
ACTIVITY 3 - OUTCOME 10

Introduce students to two methods used for finding the greatest common factor (GCF) of two numbers.

Example: Find the greatest common factor of 36 and 64.

Solution: Method 1

First, find the prime factors of 36 and 64. Use factor trees.



Next, list the factors for both numbers, and circle the factors that are in both numbers.

$$\begin{array}{l}
 36 = 2 \times 2 \times 3 \times 3 \\
 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2
 \end{array}$$

The greatest common factor of 36 and 64 is found by multiplying the prime factors that are common to both numbers ($2 \times 2 = 4$). Therefore, 4 is the GCF of 36 and 64.

Solution: Method 2

List the factors.

$$36 = 1, 2, 3, 4, 6, 9, 12, 18, 36$$

$$64 = 1, 2, 4, 8, 16, 32, 64$$

What is the largest number that is a factor of both 36 and 64? (4)

4 is the greatest common factor

ACTIVITY 4 - OUTCOME 10

Introduce students to two methods used for determining the least common multiple (LCM).

Example: Find the least common multiple of 8 and 10.

Solution: Method 1 - List the multiples.

The multiples of 8 are 8, 16, 24, 32, 40, 48 ...

The multiples of 10 are 10, 20, 30, 40, 50 ...

What is the smallest number that is a multiple of both 8 and 10? (40)

40 is the LCM for 8 and 10.

Solution: Method 2 - Use prime numbers as divisors. Several steps are involved.

Step 1: List the numbers.

8 10

Step 2: Find a prime number that will divide evenly into each of the numbers, and divide, placing the quotient below the number.

$$\begin{array}{r|rr} 2 & 8 & 10 \\ \hline & 4 & 5 \end{array}$$

Step 3: Continue this process until no prime number will divide evenly into the bottom row of numbers.

$$\begin{array}{r|rr} 2 & 8 & 10 \\ \hline & 4 & 5 \end{array}$$

Stop here. No number will go evenly in 4 and 5 except one (1).

Step 4: Multiply the divisors used and the bottom row of numbers.
 $2 \times 4 \times 5 = 40$
The product will be the LCM.

40 is the LCM.

ACTIVITY 5 - OUTCOME 10

The method previously outlined can be extended to problems asking for the LCM of 3 or more numbers.

Step 1: List the numbers.

3 6 8

Step 2: Find a prime number that will go evenly into any two of the numbers. Bring the other numbers down without dividing them.

2		3	6	8
		3	3	4
		1	1	4

Step 3: Multiply the divisors used and the digits in the bottom row. (Omit factors of 1 because they do not change the answer.)

$2 \times 3 \times 4 = 24$
24 is the LCM.

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

- OUTCOMES: 11. Recognize number patterns and relationships.
12. Develop and use number patterns in simple functions and series.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand the basic mathematical operations.

Steps to Success

1. Since some students at this level have great difficulty with this concept, teachers should begin with an explanation using words, rather than algebraic symbols. Later the teacher may want to take an algebraic approach.
2. Students should begin with examples in which the number pattern can be determined by inspection.
3. Be sure to give students a systematic approach for finding any term of a series. The need for a systematic approach occurs when the 20th, 50th, 100th, or nth term in a series is needed.

ACTIVITY 1 - OUTCOMES 11 and 12

Begin with simple examples that students can solve by inspection. Use problems such as the following:

Example: Find the next four terms in this series.

2, 4, 6, 8, ____, ____, ____, ____.

ACTIVITY 1 - OUTCOMES 11 and 12 (Cont'd)

Let students verbalize their methods for determining that the next four terms in the series would be 10, 12, 14, 16. Some students may say that they simply added 2 each time. Other students may say that they continued to list even numbers in order.

Have students generate examples to exchange with their classmates.

ACTIVITY 2 - OUTCOMES 11 and 12

The purpose of this activity is to generate a need for a systematic approach to problems involving numbers in a series. Start with the same example used in Activity 1.

2, 4, 6, 8, ____, ____, ____, ____

Ask students to find the 25th term in the series. Wait as students take the time required to count by 2 until they reach the 25th term. Tell students that there is a quick way to find any term of the series. Work through the following procedure with students.

1. Look for a pattern between the numbers in the series and their position in the series.
2. The first number in the series is 2. It can be derived by multiplying its position number by 2. See if this pattern is true for the other numbers in the series.
3. The second number in the series is 4. It is also two times its position number (2) in the series. This pattern continues with the third number in the series (6) resulting from its position in the series (3) times 2.
4. A pattern has developed. The position of the number in the series multiplied by two equals the number in the series.
5. This procedure is summarized in the following table.

Position of the number in the series	1	2	3	4	5	6 ...25 ...100
multiplied by two	x2	x2	x2	x2	x2	x2 x2 x2
equals the number in the series	2,	4,	6,	8,	<u>10,</u>	<u>12,...50,...200</u>

ACTIVITY 2 - OUTCOMES 11 and 12 (Cont'd)

Provide examples for student practice. Be sure to ask for terms in the series that require students to use a systematic approach.

Example: Find the 50th term in the following series.

5, 8, 11, 14, ...

Encourage students to set up problems in the following manner.

Position	1	2	3	4	...	50
----------	---	---	---	---	-----	----

Relationship

Terms	5,	8,	11,	14,	
-------	----	----	-----	-----	------

Wait for students to find the relationship (position multiplied by 3 added to 2 equals term). Then, the 50th term can be found ($3 \times 50 + 2 = 152$).

MATHEMATICS
GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

OUTCOME: 13. Solve horizontal equations.

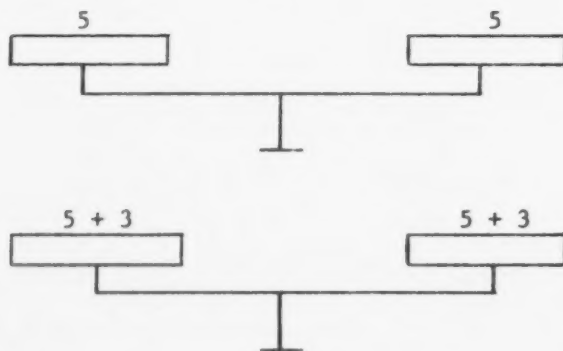
NOTES FOR THE TEACHER

Prerequisites for Students

1. Be proficient in operations with whole numbers, fractions, and decimals.
2. Understand that addition and subtraction are inverse operations and that multiplication and division are inverse operations.

Steps to Success

Students will benefit from thinking of an equation as a balanced scale. The content of each side of the scale may be altered in various ways, as long as the scale remains balanced with each alteration.



Same number added to each side.

Steps to Success (Cont'd)



Same number subtracted from each side.



Same number multiplied times each side.



Same number (3) added to each side and the same number (2) used as a divisor on each side.

When students are asked to solve equations (for example, $N + 4 = 11$), they should do the following two things:

- (1) Simplify by having one side with letters only. (N instead of $N + 4$). Have the other side consist of numbers only.
- (2) Keep the scale balanced by applying the inverse operation (or "undoing") to both sides of the equation.

$$\begin{aligned} N + 4 &= 11 \\ N + 4 - 4 &= 11 - 4 \\ N &= 7 \end{aligned}$$

Be sure that students follow correct algebraic form even with the simplest equations. Students who begin to use correct algebraic form at this level tend to have less difficulty with equation solving.

ACTIVITY 1 - OUTCOME 13

Have students work examples that require using operations of addition, subtraction, multiplication, and division as inverse operations. Ask leading questions like those found in the solutions below.

Example 1: Solve the equation $Y + 3 = 10$.

Solution: What type of number sentence is this? (addition)
What operation will "undo" addition? (subtraction)

$Y + 3 = 10$	original equation
$Y + 3 - 3 = 10 - 3$	subtraction will "undo" addition
$Y = 7$	mathematical computation

Example 2: Solve the equation $x - 5 = 12$.

Solution: What type of number sentence is this? (subtraction)
What operation will "undo" subtraction? (addition)

$x - 5 = 12$	original equation
$x - 5 + 5 = 12 + 5$	addition will "undo" subtraction
$x = 17$	mathematical computation

Example 3: Solve the equation $3X = 12$.

Solution: What type of number sentence is this? (multiplication)
What operation will "undo" multiplication? (division)

$3X = 12$	original equation
$\frac{3X}{3} = \frac{12}{3}$	division will "undo" multiplication
$X = 4$	mathematical computation

ACTIVITY 1 - OUTCOME 13 (Cont'd)

Example 4: Solve the equation $\frac{X}{2} = 6$.

$$\frac{X}{2} = 6 \quad \text{original equation}$$

$$\frac{X}{2} \cdot 2 = 6 \cdot 2 \quad \text{multiplication will "undo" division}$$

$$X = 12 \quad \text{mathematical computation}$$

MATHEMATICS
GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

OUTCOME: 1. Order and compare decimals and fractions with unlike denominators.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Find the common denominator for unlike fractions.
2. Understand the terms "greater than" and "less than."
3. Read decimals and fractions.
4. Generate equivalent fractions and equivalent decimals.
5. Order whole numbers.

Steps to Success

1. Ordering fractions is approached by assuming that students can order whole numbers. Likewise, they can order from least to greatest .125, .211, and .512 or $\frac{125}{1000}$, $\frac{211}{1000}$, $\frac{512}{1000}$.
2. When students can correctly order decimals and fractions, they must accurately read problems to determine whether they must look for "greater than" or "less than".

ACTIVITY 1 - OUTCOME 1

If decimals have the same number of digits (.230, .220, .300, .242, .211), and if fractions have the same denominators ($\frac{4}{12}$, $\frac{10}{12}$, $\frac{8}{12}$, $\frac{6}{12}$, $\frac{3}{12}$), ordering is a simple matter. It is similar to asking students to order from least to greatest 230 dogs, 220 dogs, 300 dogs, 242 dogs, and 211 dogs. (Answer: 211 dogs, 220 dogs, 230 dogs, 242 dogs, 300 dogs).

ACTIVITY 1 - OUTCOME 1 (Cont'd)

If decimals do not have the same number of digits, generate equivalent decimals by adding zeros to the right of the last digit of the decimal until all decimal numbers have the same number of digits.

Example: Order from least to greatest .23, .22, .3, .242, .211.

Solution:		Order from Least to Greatest
.23 = .230		Third
.22 = .220		Second
.3 = .300		Fifth
.242 = .242		Fourth
.211 = .211		First

Ordered from least to greatest .211, .22, .23, .242, .3.

If fractions do not have the same denominator, generate equivalent fractions through a common denominator.

Example: Order from least to greatest $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$.

Solution:		Order from Least to Greatest
$\frac{1}{3} = \frac{4}{12}$		Second
$\frac{5}{6} = \frac{10}{12}$		Fifth
$\frac{2}{3} = \frac{8}{12}$		Fourth
$\frac{1}{2} = \frac{6}{12}$		Third
$\frac{1}{4} = \frac{3}{12}$		First

Ordered from least to greatest $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$.

ACTIVITY 2 - OUTCOME 1

Problems often take the form of the example below. There is a simple solution to these problems. After the last digit to the right of the decimal, just add zeros until all decimal numbers have the same number of digits. Then, look at the number as if no decimals were there and decide which is larger or smaller.

Example: Which of the following is greater than .4?

- (a) .0058
- (b) .114
- (c) .5
- (d) .227

Solution: $.4 = .4000$

$.0058$	$= .0058$
$.114$	$= .1140$
$.5$	$= .5000$
$.227$	$= .2270$

Think of the numbers as if no decimals were there and compare 4000 to 58, 1140, 5000, and 2270. Five thousand (5000) is larger than 4000, so .5 (c) is the correct answer.

Extend examples to include mixed decimals. For example, which of the following is greater than 2.4?

- (a) 2.005
- (b) 2.114
- (c) 2.5
- (d) 2.227

In examples like these, students can see that they may ignore the whole number (2), since it appears in the question and in all choices.

ACTIVITY 3 - OUTCOME 1

Explain to students that other strategies would apply if more than one correct answer were allowed. Provide examples similar to the one below.

Example: Which of the following are greater than 2.4?

- (a) 2.005
- (b) 1.114
- (c) 2.55
- (d) 3.02

Solution: Students can learn to eliminate (b) by examining the whole numbers. Then, by inspection, (d) is one correct answer because the whole number in 3.02 is greater than the whole number in 2.4. The procedure outlined above is then applied to 2.005, and 2.55. Students must remember, however, that $2.4 = 2.400$.

$$\begin{array}{l} 2.005 = 2.005 \\ 2.55 = 2.555 \end{array}$$

Looking at the numbers as if there were no decimals shows that 2.55 is also greater than 2.4.

ACTIVITY 4 - OUTCOME 1

Fractions can be compared easily IF the denominators are the same. In order to compare fractions with unlike denominators, find a common denominator. Then, look at the numerators to decide which is larger or smaller.

ACTIVITY 4 - OUTCOME 1 (Cont'd)

Example: Which is less than $\frac{1}{6}$?

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{8}$

(d) $\frac{1}{4}$

Solution: $\frac{1}{6} = \frac{4}{24}$ Convert other fractions to equivalent fractions with denominators of 24.

$$\frac{2}{3} = \frac{16}{24}$$

$$\frac{1}{2} = \frac{12}{24}$$

$$\frac{1}{8} = \frac{3}{24}$$

$$\frac{1}{4} = \frac{6}{24}$$

Ask which fraction has a numerator less than 4. ($\frac{3}{24}$)

Consequently, $\frac{1}{8}$ is less than $\frac{1}{6}$.

Be sure to extend this activity to include mixed fractions.

ACTIVITY 4 - OUTCOME 1 (Cont'd)

Example: Which is greater than $3\frac{1}{2}$?

(a) $3\frac{5}{8}$

(b) $3\frac{1}{8}$

(c) $3\frac{1}{4}$

(d) $2\frac{7}{8}$

Solution: Eliminate $2\frac{7}{8}$ (d) because of the whole number. Then generate equivalent fractions with common denominators. $3\frac{1}{2} = 3\frac{4}{8}$

		Order from Least to Greatest
$3\frac{1}{2} = 3\frac{4}{8}$ or $\frac{28}{8}$		<u>Third</u>
$3\frac{5}{8} = 3\frac{5}{8}$ or $\frac{29}{8}$		Fourth
$3\frac{1}{8} = 3\frac{1}{8}$ or $\frac{25}{8}$		First
$3\frac{1}{4} = 3\frac{2}{8}$ or $\frac{26}{8}$		Second

Consequently, $3\frac{5}{8}$ is greater than $3\frac{1}{2}$.

MATHEMATICS
GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

OUTCOMES: 2. Review decimal place value to thousandths.
3. Review decimal place value to ten-thousandths.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand basic principles of place value.
2. Demonstrate mastery of place values using whole numbers.

Steps to Success

1. Review whole number place value.
2. Discuss the function of the decimal point (separates whole numbers from fractional numbers).
3. Build on the place value names for all place values to the left of the decimal. The place value names to the right of the decimal will add "ths" to the place value names found on the left of the decimal.

ACTIVITY 1 - OUTCOMES 2 and 3

Review place value of whole numbers using the chart below and several examples.

MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES

See activities outlined in Area Numbers, Numeration, Number Theory - Outcomes 1, 2, and 3.

Then continue this activity with a number in which all digits lie to the right of the decimal point. Be certain that students fully realize that this number will have a value less than one (fractional value).

Example: .2345

Ask students to identify the place value of each digit.

- Solution: 1. The digit in the first place to the right of the decimal is in the tenths place. Therefore, the digit 2 is in the tenths place and has a fractional value of $\frac{2}{10}$.
2. The digit in the second place to the right of the decimal is in the hundredths place. Therefore, the digit 3 is in the hundredths place and has a fractional value of $\frac{3}{100}$.
3. The digit in the third place to the right of the decimal is in the thousandths place. Therefore, the 4 is in the thousandths place and has a fractional value of $\frac{4}{1,000}$.

ACTIVITY 1 - OUTCOMES 2 and 3 (Cont'd)

4. The digit in the fourth place to the right of the decimal is in the ten-thousandths place.

Therefore, the 5 is in the ten-thousandths place and has a fractional value of $\frac{5}{10,000}$.

Continue use of place value chart by adding decimals to numbers used as examples.

MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS
5,	6	9	2,	3	4	5	• 3	2	1	5

MATHEMATICS
GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

OUTCOME: 4. Add, subtract, multiply, and divide decimals and fractions.

AREA: APPLICATIONS

OUTCOME: 1. Solve word problems about realistic situations involving whole numbers, fractions, and/or decimals requiring the use of one or more operations.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Perform basic operations with whole numbers.
2. Read, write, and understand money values.
3. Find common denominators for unlike fractions.
4. Simplify fractions.

Steps to Success

The addition, subtraction, multiplication, and division of fractions and decimals was introduced in grades 4-6. Consequently, the activities that follow are more like review activities than initial instruction. Because of the number of different examples needed to indicate mastery of this outcome, explanations are brief. Any tip or strategy is provided at the time that the type of problem is discussed. For more extensive coverage, consult Basic Skills Instructional Materials, Grade 6 and Basic Skills Instructional Materials for Alabama High School Graduation Examination.

ACTIVITY 1 - FRACTIONS AND DECIMALS - OUTCOME 4

ADDITION AND SUBTRACTION OF DECIMAL NUMBERS

When adding or subtracting decimal numbers, write the problem in vertical format. Align digits with like values under each other. After the problems are written with the decimals in a column, the rules for addition and subtraction of whole numbers will apply.

Example 1: Add $0.038 + 9 + 0.21$

Solution: Put a decimal point after the 9. Then write the problem in vertical form, align the decimal points under each other, and add.

$$\begin{array}{r} 0.038 \\ 9. \\ + 0.21 \\ \hline 9.248 \end{array} \text{ is the answer.}$$

Problems containing money values should be worked in the same way.

Example 2: Add $\$42 + \$3.45 + \$0.75$

Solution: Put a decimal point after the 2 in \$42. Repeat the process explained in Example 1.

$$\begin{array}{r} \$42. \\ 3.45 \\ + .75 \\ \hline \$46.20 \end{array} \text{ is the answer.}$$

Example 3: Subtract $3 - .004$

Solution: Put a decimal after the 3. Then write the problem in vertical form, aligning the decimal points under each other. Add three zeros after the 3 and subtract.

$$\begin{array}{r} 3. \quad 3.000 \quad \begin{array}{r} 299 \\ 3.000 \\ - .004 \\ \hline 2.996 \end{array} \\ - .004 \quad - .004 \quad - .004 \\ \hline \end{array}$$

ACTIVITY 2 - FRACTIONS AND DECIMALS - OUTCOME 4

MULTIPLICATION OF DECIMAL NUMBERS

Multiply decimal numbers exactly as if they were whole numbers. When a product is obtained, go back to the original problem and count the total number of digits to the right of the decimals in both factors. Begin at the digit farthest to the right in the product, and count back to the left the same number of places as was obtained when the digits to the right of the decimals were counted in the factors (previous step). Place the decimal at that point.

Example: Multiply $8.14 \times .6$

Solution: First, ignore the decimal points and multiply just as though no decimal points were there.

$$\begin{array}{r} 8.14 \\ \times .6 \\ \hline 4884 \end{array}$$

Next: Count the total number of digits to the right of the decimal points in both factors.

8.14 has 2 digits after the decimal and $.6$ has 1 digit after the decimal.

Therefore, count 3 places starting at the digit farthest to the right in the product and place the decimal point.

$$\begin{array}{r} 8.14 \\ \times .6 \\ \hline 4.884 \end{array}$$

4.884 is the answer.

ACTIVITY 3 - FRACTIONS AND DECIMALS - OUTCOME 4

DIVISION OF DECIMAL NUMBERS

There are two important things to remember when dividing decimals.

First, always divide by a whole number.

ACTIVITY 3 - FRACTIONS AND DECIMALS - OUTCOME 4 (Cont'd)

"Always divide by a whole number" means that if there is a decimal in the divisor, then the problem must be transformed to an equivalent problem which contains no decimal in the divisor.

$$.32 \overline{)1.234} \quad \text{can be transformed into} \quad 32 \overline{)123.4}$$

Explain that division problems can be transformed into equivalent problems by multiplying the dividend and divisor by the same number.

$$.32 \times 100 = 32 \quad 1.234 \times 100 = 123.4$$

Be sure that students understand that moving decimal points in the divisor and dividend the same number of places to the right is equivalent to multiplying both the divisor and dividend by the same number. Emphasize that the value of the problem is not changed by multiplying both the divisor and dividend by the same number.

$$.32 \overline{)1.234} \quad 32 \overline{)123.4}$$

Second, always place the decimal point in the answer (quotient) directly above the decimal point in the number being divided (dividend).

$$32 \overline{)123.4}$$

Example 1: Divide $.24 \overline{)14.64}$

Solution: (1) Change the divisor into a whole number.

$$\cancel{.24} \overline{)14.64} \quad 24. \overline{)1464.}$$

(2) Place the decimal in the answer before dividing.

$$24. \overline{)1464.}$$

(3) The answer is 61.

$$\begin{array}{r} 61. \\ 24. \overline{)1464.} \\ \underline{144} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

ACTIVITY 3 - FRACTIONS AND DECIMALS - OUTCOME 4 (Cont'd)

Sometimes it is necessary to add zeros in order to place the decimal point. Remember that zeros can be added after the decimal point without changing the value of the number.

Example 2: Divide $1.2 \overline{)9}$

Solution: (1) Change the divisor into a whole number. To do this, place a decimal after the 9 and add zeros as needed.

$$1.2 \overline{)9.0} \qquad 12 \overline{)90.}$$

(2) Place the decimal in the answer before dividing.

$$12 \overline{)90.}$$

(3) Perform the operation.

$$\begin{array}{r} 7.5 \\ 12 \overline{)90.0} \\ \underline{84} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

(4) 7.5 is the answer.

ACTIVITY 4 - FRACTIONS AND DECIMALS - OUTCOME 4 APPLICATIONS - OUTCOME 1

If the fractions to be added or subtracted have the same denominators, then add or subtract the numerators to find the answer. Finding a common denominator is the first step in adding and subtracting fractions with unlike denominators. The last step in all problems involving fractions is to write the answer in simplest (lowest) terms.

Example 1: Jill bought $3 \frac{1}{2}$ pounds of ground beef and $2 \frac{3}{4}$ pounds of chicken. What was the total weight of her purchase?

ACTIVITY 4 - FRACTIONS AND DECIMALS - OUTCOME 4 (Cont'd)
APPLICATIONS - OUTCOME 1

Solution:

$$\begin{array}{r} 3 \frac{1}{2} = 3 \frac{2}{4} \\ + 2 \frac{3}{4} = 2 \frac{3}{4} \\ \hline 5 \frac{5}{4} \end{array}$$

Even though $5 \frac{5}{4}$ is a correct answer, it is not written in its simplest form.

Write $5 \frac{5}{4}$ in its simplest form.

$$5 \frac{5}{4} = 5 + 1 \frac{1}{4} = 6 \frac{1}{4}.$$

Example 2: Mr. Green weighs 162 pounds. Mrs. Green weighs $118 \frac{3}{4}$ pounds. Find the difference between their weights.

Solution:

$$\begin{array}{r} 162 \text{ lb.} \\ - 118 \frac{3}{4} \text{ lb.} \\ \hline \end{array}$$

Borrow 1 from 162 and write it as the fraction $\frac{4}{4}$.

$$\begin{array}{r} 161 \frac{4}{4} \text{ lb.} \\ - 118 \frac{3}{4} \text{ lb.} \\ \hline 43 \frac{1}{4} \text{ lb.} \end{array}$$

The denominators are alike,
so subtract.

ACTIVITY 5 - FRACTIONS AND DECIMALS - OUTCOME 4

- (1) To multiply fractions, multiply the numerators and then multiply the denominators.
- (2) Multiply a whole number times a fraction by changing the whole number to a fraction with a denominator of 1 such as $5 = \frac{5}{1}$, $2 = \frac{2}{1}$, $3 = \frac{3}{1}$, etc. Then multiply the fractions as described in step 1.
- (3) To multiply a mixed number by a fraction, change the mixed number to an improper fraction. Then multiply the fractions as described in Step 1.
- (4) The last step in any problem involving fractions is to simplify the answer. Simplifying answers means to reduce all fractions to lowest terms and change improper fractions to mixed numbers.

Example 1: Larry worked $\frac{3}{4}$ of an hour each day for 5 days. How many hours did he work in all?

Solution: Write 5 as a fraction $\frac{5}{1}$ and multiply.

$$\frac{5}{1} \times \frac{3}{4} = \frac{5 \times 3}{1 \times 4} = \frac{15}{4} \text{ hours}$$

$$\text{Simplify: } \frac{15}{4} = 3 \frac{3}{4} \text{ hours}$$

Example 2: Joyce worked $2 \frac{1}{4}$ hours a day for 6 days. How many hours did she work in all?

Solution: Write $2 \frac{1}{4}$ as an improper fraction. Also, write 6 as a fraction. Then, multiply.

$$2 \frac{1}{4} = \frac{9}{4} \quad \text{and} \quad 6 = \frac{6}{1}$$

ACTIVITY 5 - FRACTIONS AND DECIMALS - OUTCOME 4 (Cont'd)

$$\frac{9}{4} \times \frac{6}{1} = \frac{54}{4} \text{ hours}$$

$$\text{Simplify: } \frac{54}{4} = 13 \frac{2}{4} = 13 \frac{1}{2} \text{ hours}$$

ACTIVITY 6 - FRACTIONS AND DECIMALS - OUTCOME 4

To divide a fraction, multiply by the reciprocal of the divisor. Be sure to include examples such as the following:

Example 1: $\frac{3}{5} \div \frac{1}{4} =$

$$\frac{3}{5} \times \frac{4}{1} = \frac{12}{5} \text{ Find the reciprocal of the divisor and multiply.}$$

$$\frac{12}{5} = 2 \frac{2}{5} \text{ Simplify.}$$

Example 2: $\frac{3}{4} \div 2 =$

$$\frac{3}{4} \div \frac{2}{1} = \text{Change the 2 to a fraction.}$$

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8} \text{ Find the reciprocal of the divisor and multiply.}$$

ACTIVITY 6 - FRACTIONS AND DECIMALS - OUTCOME 4 (Cont'd)

Example 3: $2 \frac{1}{2} \div \frac{3}{4} =$

$$\frac{5}{2} \div \frac{3}{4} =$$

Change the mixed number to an improper fraction.

$$\frac{5}{2} \times \frac{4}{3} =$$

Find the reciprocal of the divisor.

$$\frac{5}{\cancel{2}} \times \frac{\cancel{4}}{3} = \frac{10}{3}$$

Simplify by cancellation and multiply.

$$\frac{10}{3} = 3 \frac{1}{3}$$

Simplify.

Example 4: $1 \frac{4}{5} \div 1 \frac{2}{3} =$

$$\frac{9}{5} \div \frac{5}{3} =$$

Change the mixed numbers to improper fractions.

$$\frac{9}{5} \times \frac{3}{5} = \frac{27}{25}$$

Find the reciprocal of the divisor and multiply.

$$\frac{27}{25} = 1 \frac{2}{25}$$

Simplify.

MATHEMATICS
GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

- OUTCOMES:
5. Solve problems using decimals larger than one and mixed numbers.
 6. Solve problems with metric and money values.
 7. Solve word problems involving decimals and fractions.
 8. Solve word problems involving decimals, fractions, and ratios and proportions.
 9. Solve problems involving addition, subtraction, and multiplication of fractions.

AREA: APPLICATIONS

- OUTCOME:
1. Solve word problems about realistic situations involving whole numbers, fractions, and/or decimals requiring the use of one or more operations.

NOTES FOR THE TEACHER:

Prerequisites for Students

1. Use the basic operations on whole numbers, decimals, and fractions.
2. Recognize ratio and proportion word problems, and have practice in solving them.

Steps to Success

Students need to develop a procedure for solving word problems. They will need to learn to read carefully and to look for "key" words that indicate which operations are appropriate. The following steps will assist in solving word problems:

1. Read the problem.
2. What is asked for?
3. What is known?
4. What operation and/or operations are needed?
5. Show work.
6. Find the answer.
7. Is the answer reasonable?

ACTIVITY 1 - FRACTIONS AND DECIMALS - OUTCOMES 5, 7, and 8
APPLICATIONS - OUTCOME 1

Select a series of problems involving decimals numbers larger than one and mixed numbers. Work a few problems with students demonstrating the steps previously outlined.

Example: John took a trip. On Monday he traveled 50.4 miles, on Tuesday he traveled 200.25 miles, and on Wednesday he traveled 175.5 miles. How far did he travel on his trip?

Solution:	<u>STEP</u>	<u>ANSWER</u>
1.	Read the problem.	
2.	What is asked for?	2. How far John traveled
3.	What is known?	3. Monday - 50.4 miles Tuesday - 200.25 miles Wednesday - 175.5 miles
4.	What operation is needed?	4. Addition
5.	Show work.	5. $\begin{array}{r} 50.40 \\ 200.25 \\ + 175.50 \\ \hline 426.15 \end{array}$
6.	Find answer.	6. 426.15 miles
7.	Is answer reasonable?	7. Yes. The answer should be a little less than $50 + 200 + 200$ or less than 450.

ACTIVITY 2 - FRACTIONS AND DECIMALS - OUTCOMES 5, 7, and 8
APPLICATIONS - OUTCOME 1

When students have worked several examples like this, have them make up examples. Give precise specifications for problems.

Specifications - Create word problems involving _____.
(mixed fractions or decimal numbers larger than one)

Make the problem a/an _____ problem.
(addition, subtraction, multiplication, division).

A large number of different problems is possible. Have students exchange problems, verify whether or not the problems meet the teacher's specifications, and solve the problems. An example would be:

Specifications - Create a word problem involving mixed fractions.
Make it a subtraction problem.

Example: Jane walked $2\frac{1}{2}$ miles. Pam walked $3\frac{1}{4}$ miles. How much farther did Pam walk than Jane?

ACTIVITY 3 - FRACTIONS AND DECIMALS - OUTCOMES 6, 7, and 8
APPLICATIONS - OUTCOME 1

Extend examples to include those involving metric and money values.

Example: John purchased a tie for \$5.25, a shirt for \$15, and a pair of socks for \$2. He gave the clerk \$50.00. How much change should he receive if he did not have to pay sales tax on this purchase?

Solution: STEP

ANSWER

1. Read the problem.

2. What is asked for?

2. How much change John will receive

ACTIVITY 3 - FRACTIONS AND DECIMALS - OUTCOMES 6, 7, and 8 (Cont'd)
APPLICATIONS - OUTCOME 1

<u>Solution:</u>	<u>STEP</u>	<u>ANSWER</u>
3.	What is known?	3. The tie cost \$5.25. The shirt cost \$15. The socks cost \$2. He gave the clerk \$50. No sales tax was charged on the purchase.
4.	What operation/operations are needed?	4. (1) Add to find the total cost of the items. (2) Subtract to find the change.
5.	Show work.	5. $\begin{array}{r} \$ 5.25 \\ 15.00 \\ + 2.00 \\ \hline \$ 22.25 \end{array}$ Total cost $\begin{array}{r} \$ 50.00 \\ - 22.25 \\ \hline \$ 27.75 \end{array}$ Amount given to clerk Amount of purchase Change
6.	Find the answer.	6. John received \$27.75 change.
7.	Is the answer reasonable?	7. Yes. John spent approximately \$25. He will receive approximately \$25 in change.

ACTIVITY 4 - FRACTIONS AND DECIMALS - OUTCOMES 6, 7, and 8

Extend the exercise of students making up their own problems to meet the teacher's specifications (Activity 2).
 Extend specifications to include problems using metric and/or money values.

Specifications - Create word problems involving _____.
 (mixed numbers, decimals, money, metric values).

Make the problem a/an _____ problem.
 (addition, subtraction, multiplication, division).

Example: A piece of lumber is 10 meters long. How many boards of 2.5 meters each can be cut from the lumber?

ACTIVITY 5 - FRACTIONS AND DECIMALS - OUTCOMES 7, 8, and 9

Again, have students exchange problems, verify compliance with the teacher's specifications, and solve the problems.

Be sure that teacher examples and student problems include work with fractions.

Example 1: Susie plans to make 5 pairs of curtains for her house. Each pair of curtains requires $6\frac{1}{2}$ yards of material. How much material should Susie purchase?

Example 2: Mrs. Smith, the cheerleader sponsor, ordered $16\frac{1}{4}$ yards of material to make 5 cheerleader uniforms. How much material does it take to make one uniform?

ACTIVITY 6 - FRACTIONS AND DECIMALS - OUTCOME 8

Teacher examples and student problems can also include work with ratios and proportions.

Example: There are 10 computers for every 100 students in the local school. If the enrollment of the school is 500 students, how many computers does the school have?

Solution:	<u>STEP</u>	<u>ANSWER</u>
	1. Read the problem.	
	2. What is asked for?	2. How many computers are in the school?
	3. What is known?	3. There are 10 computers for every 100 students. Current enrollment = 500
	4. What operation is needed?	4. Ratio and proportion.

ACTIVITY 6 - FRACTIONS AND DECIMALS - OUTCOME 8 (Cont'd)

5. Show work.

$$5. \quad \frac{\text{computers}}{\text{enrollment}} = \frac{\text{computers}}{\text{enrollment}}$$

$$\frac{10}{100} = \frac{N}{500}$$

$$100 N = 5000$$

$$N = 50$$

6. Find answer.

6. 50 computers

7. Is answer reasonable?

7. Yes.

$$500 = 5 \times 100$$

$$50 = 5 \times 10$$

MATHEMATICS
GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

OUTCOME: 12. Write equivalent fractions.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Use basic operations of multiplication and division.
2. Have a basic understanding of fractions including finding common denominators.

Steps to Success

1. Emphasize the meaning of equivalent.
2. Refer to several examples where equivalent expressions are used in mathematics.
3. Show students several methods of proving fractions equivalent.

ACTIVITY 1 - OUTCOME 12

Begin by introducing students to the word "equivalent." Explain that they may have used this word in previous work, or they may have used the "is equal to" terminology.

Show that $3 + 2 = 5 = 4 + 1$

Emphasize that $3 + 2$ is not the same as 5 nor the same as $4 + 1$; however, $3 + 2$ is "equal to" or "equivalent to" 5 and $4 + 1$. Equivalent simply means equal in quantity or value.

ACTIVITY 1 - OUTCOME 12 (Cont'd)

Illustrate equivalent expressions by using coins and bills (money) as examples. Use actual coins, or summarize what students already know in a table like the one below.

COIN	VALUE IN RELATION TO \$ 1	NUMBER OF COINS NEEDED TO MAKE \$ 1	EQUIVALENT VALUES
Half-dollar Fifty cent piece	$\frac{1}{2} \times \$ 1$	$2 \times \frac{1}{2}$ dollar	$2 \times \frac{1}{2} = \frac{2}{2} = 1$
Quarter	$\frac{1}{4} \times \$ 1$	$4 \times \frac{1}{4}$ dollar	
Dime Ten cents		$10 \times \frac{1}{10}$ dollar	$10 \times \frac{1}{10} = \frac{10}{10} = 1$
Penny One cent	$\frac{1}{100} \times \$ 1$		$100 \times \frac{1}{100} = \frac{100}{100} = 1$

Work out the table above with students. Make sure they understand what should be written in each location. Leave blanks to see if they can complete the table correctly.

From the fourth column in the table (Equivalent Values) students can observe something about equivalent expressions:

$$1 = 2 \times \frac{1}{2} = \frac{2}{2} = 4 \times \frac{1}{4} = \frac{4}{4} = 10 \times \frac{1}{10} = \frac{10}{10} = 100 \times \frac{1}{100} = \frac{100}{100}$$

Students can also observe something about equivalent fractions:

$$\frac{1}{1} = \frac{2}{2} = \frac{4}{4} = \frac{10}{10} = \frac{100}{100}$$

Emphasize that fractions which are not alike in appearance can have the same value.

ACTIVITY 2 - OUTCOME 12

Start with the following discovery from Activity 1:

$$\frac{1}{1} = \frac{2}{2} = \frac{4}{4} = \frac{10}{10} = \frac{100}{100}$$

Ask what was done to the first fraction $\left(\frac{1}{1}\right)$ to get the other fractions. (The numerators and denominators of each equivalent fraction were multiplied by a specific number.

$$\frac{1}{1} = \frac{1 \times 2}{1 \times 2} = \frac{2}{2}$$

$$\frac{1}{1} = \frac{1 \times 4}{1 \times 4} = \frac{4}{4}$$

$$\frac{1}{1} = \frac{1 \times 100}{1 \times 100} = \frac{100}{100}$$

Emphasize that any number times one (1) equals that same number. Therefore:

$$3 \times 1 = 3 \text{ or } 3 \times \frac{2}{2} = 3 \text{ or } 3 \times \frac{100}{100} = 3$$

$$N \times 1 = N \text{ or } N \times \frac{2}{2} = N \text{ or } N \times \frac{100}{100} = N$$

Consequently, any fraction has an infinite number of equivalent fractions generated by multiplying both numerator and denominator by successive numbers.

$$\frac{1}{2} = \frac{1 \times 1}{2 \times 1} = \frac{1 \times 2}{2 \times 2} = \frac{1 \times 3}{2 \times 3} = \frac{1 \times 4}{2 \times 4} = \frac{1 \times 5}{2 \times 5} \dots$$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} \dots$$

ACTIVITY 2 - OUTCOME 12 (Cont'd)

Have students practice generating families of equivalent expressions:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} \dots$$

$$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} \dots$$

Extend the activity of generating families of equivalent fractions to include families with missing members.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \dots = \frac{\quad}{20} = \dots = \frac{\quad}{50} = \dots = \frac{\quad}{100}$$

For any missing member (for example $\frac{\quad}{20}$), ask students what was done to the denominator of the first fraction ($\frac{1}{2}$) to generate and the denominator of the missing member. Students can observe that it was multiplied by 10.

Consequently, they must multiply the numerator by 10 to keep the fractions equivalent.

$$\frac{1}{2} = \frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

ACTIVITY 3 - OUTCOME 12

Sometimes students are asked to find only one "family member" in the family of equivalent fractions.

Example: $\frac{3}{4} = \frac{\quad}{8}$ or $\frac{2}{3} = \frac{\quad}{6}$

Solution: Ask students what was done to the denominator in the first fractions to get the denominator in the second fractions (multiplied by 2). To find the numerator of the second fractions, multiply the numerator of the first fractions by 2 also.

Point out that examples like these are required when common denominators are needed to solve addition and subtraction problems involving some fractions.

ACTIVITY 4 - OUTCOME 12

Students are often asked to determine if two fractions are equivalent. Two procedures can be used to answer this question.

Example: Are the following fractions equivalent?

$$\frac{2}{5} \quad \text{and} \quad \frac{10}{25}$$

Solution 1: Change the fractions to fractions with like denominators:

$$\frac{2}{5} = \frac{10}{25}$$

$$\frac{10}{25} = \frac{10}{25} \quad \text{Therefore, } \frac{2}{5} \text{ and } \frac{10}{25} \text{ are equivalent fractions.}$$

Solution 2: Solve the problem by using cross products, remembering that if cross products are equal, the fractions are equal.

$$\text{Does } \frac{2}{5} = \frac{10}{25}$$

$$\frac{2}{5} \nearrow \frac{10}{25}$$

and

$$\frac{2}{5} \searrow \frac{10}{25}$$

$$5 \times 10 = 50$$

and

$$2 \times 25 = 50$$

Since the cross products are equal, $\frac{2}{5}$ and $\frac{10}{25}$ are equivalent fractions.

ACTIVITY 4 - OUTCOME 12 (Cont'd)

Example: Are the following fractions equivalent?

$$\frac{3}{5} \text{ and } \frac{10}{25}$$

Solution:

$$\frac{3}{5} \begin{array}{c} \nearrow \searrow \\ \searrow \nearrow \end{array} \frac{10}{25}$$

$$5 \times 10 = 50 \quad 3 \times 25 = 75$$

$$50 \neq 75$$

Therefore, $\frac{3}{5}$ and $\frac{10}{25}$ are not equivalent.

MATHEMATICS
GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

OUTCOME: 15. Convert fractions to decimals and vice versa.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Divide whole numbers and decimals.
2. Read a fraction in two ways: (For example $\frac{2}{5}$ can be read two fifths or two divided by five.)
3. Change fractions to equivalent fractions.
4. Recognize multiples of 100.
5. Simplify fractions.

Steps to Success

1. Converting fractions to decimals is dealt with in Activity 1.
2. Activity 2 involves conversion of decimals to fractions.
3. Activity 3 outlines some extensions.

ACTIVITY 1 - OUTCOME 15

Begin with fractions where the denominator is a multiple or factor of 10. This is the most explicit illustration of the relation between fractions and decimals. Solve the conversion problem by generating an equivalent fraction with 10, 100, 1000, 10,000, etc. as the denominator.

Example: Change $\frac{2}{5}$ to a decimal.

Solution: $\frac{2}{5} = \frac{4}{10} = \frac{40}{100} \dots$

$\frac{4}{10}$ can be written as a decimal (.4)

$\frac{40}{100}$ can be written as a decimal (.40)

Therefore $\frac{2}{5} = .4$ or .40 or .400 ...

It is important for students to understand that this procedure works only with fractions when the denominator is a factor or multiple of 10 or 100.

Examples: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}, \frac{1}{20} \dots$

Other examples require a different procedure. If the denominator is not a factor or multiple of 10 or 100, the fraction must be "read" as a division problem.

ACTIVITY 1 - OUTCOME 15 (Cont'd)

Example: Convert $\frac{1}{6}$ to a decimal.

Solution: $\frac{1}{6}$ is read "one divided by six." And the conversion is obtained through division.

$$\begin{array}{r} \frac{1}{6} = 6 \overline{) 1.000} \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

Point out that conversion of fractions to decimals works with all fractions since all fractions can be read as a division problem.

$\frac{1}{4}$ can be read "one divided by four."

$$\begin{array}{r} \frac{1}{4} = 4 \overline{) 1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Notice that fractions containing denominators which are factors or multiples of 10 and 100 will have no remainders when division is performed. All other fractions will have remainders when they are converted to decimals by division.

ACTIVITY 2 - OUTCOME 15

All decimals can be written as fractions with denominator of 10, 100, 1000, 10,000.... Therefore, converting decimals to fractions consists of:

1. reading the decimal and listening to what will be the denominator of the fraction; and
2. reducing that fraction to its lowest terms.

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ACTIVITY 2 - OUTCOME 15 (Cont'd)

Example: Write .6 as a fraction.

Solution: .6 is read "six tenths."

.6 is written $\frac{6}{10}$.

$\frac{6}{10}$ can be reduced to $\frac{3}{5}$.

ACTIVITY 3 - OUTCOME 15

When studying the conversion of fractions to decimals, teachers may wish to discuss recurring decimals and their mathematical notation.

Example: $\frac{1}{3}$ is equivalent to what decimal number?

Solution:
$$\begin{array}{r} .333333 \\ 3 \overline{) 1.00} \\ \underline{9} \\ 10 \end{array} \quad \text{or} \quad 0.\overline{3}$$

Example: Change $\frac{8}{11}$ to a decimal.

Solution:
$$\begin{array}{r} .727272 \\ 11 \overline{) 8.0000} \\ \underline{77} \\ 30 \\ \underline{22} \\ 80 \\ \underline{77} \\ 3 \end{array} \quad \text{or} \quad 0.\overline{72}$$

MATHEMATICS
GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

OUTCOME: 7. Demonstrate an understanding of percent terminology, e.g., percent, percentage, and the percent symbol (%).

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand place value.
2. Convert fractions to percents.
3. Convert percents to decimals and vice versa.

Step to Success

1. Clearly define percent terminology.

ACTIVITY 1 - OUTCOME 7

For students to be successful with problems involving percent, they must understand and use the different terms in these problems. When introducing these definitions, give as many examples as necessary to explain the terms and the different ways they may be written in the problem.

Provide students with definitions similar to the following:

1. Percent - "per hundred." Percent may be expressed as a fraction (or ratio) with a denominator of 100.
2. Rate - the number written with the percent symbol in problems involving percents.
3. Percentage - a number equal to the product of a rate and another number called the base.

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ACTIVITY 1 - OUTCOME 7 (Cont'd)

Give students several percent problems and have them identify the parts. Use the terms "rate", "base", and "percentage" as the labels.

Example 1: What is 20% of 287?

Solution: rate - 20%
 base - 287
 percentage - 20% x 287

Problems may be written in different forms.

Example 2: 20% of 287 is what number?

Solution: rate - 20%
 base - 287
 percentage - 20% x 287

MATHEMATICS
GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

OUTCOME: 8. Change percents to decimals and decimals to percents.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand and use percent terminology.
2. Write percents as fractions and generate equivalent fractions.
3. Understand the movement of decimals as it relates to the multiplication and division of powers of 10.

Steps to Success

1. Review percent terminology.
2. Review equivalent fractions.

ACTIVITY 1 - OUTCOME 8

Students usually remember that in changing percents to decimals or decimals to percents, the decimal moves two places to the right or two places to the left. They often forget which direction the decimal moves in each operation.

Emphasize that percent is defined as per hundred or as a ratio with 100 as a denominator. Therefore, conversion from percents to decimals involves writing the ratio as the numerator of a fraction (omitting the percent symbol) over the denominator of 100. Then the division rule of powers of 10 applies in writing the fraction as a decimal.

ACTIVITY 1 - OUTCOME 8 (Cont'd)

Example 1: Change 25% to a decimal.

Solution: Ask leading questions such as the following:

1. What does the percent symbol (%) mean? (per hundred)
2. Write 25% as a fraction. $\left(\frac{25}{100}\right)$
3. Write $\frac{25}{100}$ as a decimal. (.25)

After several examples similar to the one above, extend the discussion by asking leading questions similar to the following:

1. What happened when $\frac{25}{100}$ was converted to a decimal?
(Only the numerator was written, and the decimal was moved two places to the left.)
2. Why was the decimal moved two places to the left? (25 was divided by 100 or 10^2 .)
3. Ask students to verbalize the relationship between the movement of decimals and division by 100 (the second power of 10).

When students have understood problems such as Example 1, provide problems in which the percent involves a decimal.

Example 2: Change 15.5% to a decimal.

Solution: In this problem a decimal point is already written in the problem. Using the procedure described above, move the decimal point 2 places to the left and drop the "%" symbol. Therefore, $15.5\% = 0.155$.

ACTIVITY 2 - OUTCOME 8

By reversing the process described in Activity 1, students can also change decimals to percents.

ACTIVITY 2 - OUTCOME 8 (Cont'd)

Example 1: Change .36 to a percent.

- Solution:
1. Write .36 as a fraction? $\frac{36}{100}$
 2. Write $\frac{36}{100}$ as a percent. (36%)

Example 2: Change .5 to a percent.

- Solution:
1. Write .5 as a fraction. $\frac{5}{10}$
 2. Change $\frac{5}{10}$ to an equivalent fraction with a denominator of 100. $\frac{5}{10} = \frac{50}{100}$
 3. Write $\frac{50}{100}$ as a percent. (50%)

Use examples similar to these to develop the rule: To change decimals to percents, without converting to fractions, move the decimal point 2 places to the right and add the "%" symbol to the answer.

Example 3: Change 4.5 to a percent.

- Solution:
1. Move the decimal point two places to the right. 4.5 450.
 2. Add the % symbol. 4.5 = 450%

MATHEMATICS
GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

- OUTCOMES:
1. Write and use ratios to compare one quantity to another.
 2. Write a proportion using two equal ratios.
 3. Solve proportions.
 4. Estimate solutions to proportions.
 5. Use proportions to solve word problems.
 6. Use proportions to solve word problems involving rates or scale drawings.
 9. Recognize percent as a ratio.

AREA: FRACTIONS AND DECIMALS

- OUTCOMES:
13. Recognize fractions as ratios and equivalent fractions as proportions.
 14. Use equivalent fractions to solve proportion problems. Use both ratio symbols.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand equivalent fractions, estimations, and equalities.
2. Simplify fractions.
3. Solve simple equations that are written in the form $ax = b$.

Step to Success

1. Review changing fractions to equivalent fractions.

ACTIVITY 1 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1 and 9
FRACTIONS AND DECIMALS - OUTCOME 13

Explain that a ratio is a method of comparing two numbers. Give examples similar to those below.

ACTIVITY 1 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1 and 9 (Cont'd)
FRACTIONS AND DECIMALS - OUTCOME 13 (Cont'd)

Example 1: There are 12 boys and 15 girls in the math class. What is the ratio of boys to girls?

Solution: The ratio may be expressed in different ways:

$$\frac{12}{15} \text{ or } 12 \text{ to } 15 \text{ or } 12:15 \text{ or } 0.8 \text{ or } 80\%$$

Explain that the numbers 12 and 15 are called the terms of the ratio.

Notice that the question is, "What is the ratio of boys to girls?". This gives the clue for which term is written as the numerator and which term is written as the denominator. The first term written (boys) is the numerator of the fraction. The second term written (girls) is the denominator of the fraction.

Example 2: Take the same example, but change the question.

There are 12 boys and 15 girls in a math class. What is the ratio of girls to boys?

Solution: Notice the change in the question. Now the number of girls (15) becomes the numerator and the number of boys (12) becomes the denominator.

The ratio may be expressed as $\frac{15}{12}$ or 15:12 or 1.25 or 125%.

Often, the problem requires that the ratio be written in simplest form. In this case, the ratio $\frac{15}{12}$ could be simplified to $\frac{5}{4}$. By definition, however, a ratio is a fraction. Therefore,

$\frac{5}{4}$ would not be written as a mixed number $1 \frac{1}{4}$.

ACTIVITY 2 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1 and 2
FRACTIONS AND DECIMALS - OUTCOME 14

Ratios that have the same value are called equal (or equivalent) ratios. Equivalent fractions are equal ratios.

$\frac{2}{3}$ and $\frac{4}{6}$ are equal or equivalent ratios. $\frac{5}{8}$ and $\frac{10}{16}$ are equal or equivalent ratios.

ACTIVITY 2 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1 and 2 (Cont'd)
FRACTIONS AND DECIMALS - OUTCOME 14 (Cont'd)

A statement showing that two ratios are equal is called a proportion. $\frac{2}{3} = \frac{4}{6}$ is a proportion. $\frac{5}{8} = \frac{10}{16}$ is also a proportion.

Example 1: Complete this proportion. $\frac{5}{6} = \frac{?}{12}$

Solution: Since $6 \times 2 = 12$, multiply the numerator (5) by 2 to determine the unknown numerator.

$$\begin{array}{r} 5 \times 2 = 10 \\ 6 \times 2 = 12 \\ \hline \text{Therefore, } \frac{5}{6} = \frac{10}{12}. \end{array}$$

Example 2: Are these ratios equal? Write = or $\neq$.

(a) $\frac{2}{3} \square \frac{4}{9}$

(b) $\frac{3}{4} \square \frac{12}{16}$

Solution: Several methods may be used to solve these examples. Remind students that a proportion is a statement in which two ratios are equal. Ask the students to recall methods for proving that fractions are equivalent.

(a) Method 1: Determine equality or inequality of ratios by changing fractions to equivalent fractions with like denominators.

Ask students if $\frac{2}{3} = \frac{4}{9}$ or if $\frac{2}{3} \neq \frac{4}{9}$

Change fractions to equivalent fractions with like denominators. $\left(\frac{2}{3} = \frac{6}{9}\right)$

$\frac{2}{3} \neq \frac{4}{9}$ because $\frac{2}{3} = \frac{6}{9}$

ACTIVITY 2 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1 and 2 (Cont'd)
FRACTIONS AND DECIMALS - OUTCOME 14 (Cont'd)

Method 2: Determine equality or inequality of ratios by finding whether or not their cross-products are equal.

Ask students if $\frac{2}{3} = \frac{4}{9}$ or if $\frac{2}{3} \neq \frac{4}{9}$

Multiply to determine the cross-products ($3 \times 4 = 12$ and $2 \times 9 = 18$)

$\frac{2}{3} \neq \frac{4}{9}$ because $12 \neq 18$

Use Method 1 to show that $\frac{3}{4} =$ or $\neq \frac{12}{16}$

$$\frac{3}{4} = \frac{\quad}{16}$$

$$\frac{3}{4} = \frac{3 \times 4}{16} \text{ and therefore } \frac{3}{4} = \frac{12}{16}$$

Use Method 2 to show that $\frac{4}{5} =$ or $\neq \frac{7}{10}$

$$\frac{4}{5} \begin{array}{c} \nearrow 7 \\ \searrow 10 \end{array}$$

$$4 \times 10 = 5 \times 7 \\ 40 \neq 35 \text{ and therefore } \frac{4}{5} \neq \frac{7}{10}$$

ACTIVITY 3 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1, 2, and 3

Use Method 1 and Method 2 to solve simple proportion problems. Give examples similar to the following.

Example: Solve the proportion: $\frac{5}{9} = \frac{n}{18}$

Method 1: Since $\frac{5 \times 2}{9 \times 2} = \frac{10}{18}$ then n must be replaced with 10 to make the proportion equal.

Method 2: $\frac{5}{9} = \frac{n}{18}$

Students may find it easier to write the cross-product containing the variable (9n) on the left side of the equation.

$$\frac{5}{9} = \frac{n}{18}$$

$$9n = 5 \times 18$$

$$9n = 90$$

$$n = 10$$

Check by using cross-products

$$\frac{5}{9} = \frac{10}{18}$$

$$90 = 90$$

Therefore, $\frac{5}{9} = \frac{10}{18}$ is a proportion.

ACTIVITY 4 - RATIO, PROPORTION, AND PERCENT - OUTCOME 4

Example: The local supermarket advertised candy bars 2 for \$.99. John purchased 6 candy bars. The clerk charged him \$4.00. Estimate if John's bill was correct.

Solution: John did not have paper or pencil. Mentally, he thought, "\$.99 is almost \$1.00. I can use a proportion, and if the cross-products are equal, my bill is correct."

$$\frac{2}{\$1} = \frac{6}{\$4}$$

ACTIVITY 4 - RATIO, PROPORTION, AND PERCENT - OUTCOME 4 (Cont'd)

Mentally, he checked the accuracy of the proportion by cross-products.

$$\frac{2}{1} = \frac{6}{4}; \quad 8 \neq 6. \quad \text{Therefore, John's bill was not correct.}$$

ACTIVITY 5 - RATIO, PROPORTION, AND PERCENT - OUTCOMES 1, 2, 3, 4, and 5

In writing proportions, the most important consideration is the correct placement of the terms. Use examples similar to the ones below to provide practice for students.

Example 1: A school has 2 teachers for every 60 students. There are 300 students in the school. How many teachers are there?

Solution: Emphasize that when writing a proportion, the location of the terms in the ratios is important. In this problem the proportion should be written:

$$\frac{\text{teachers}}{\text{students}} = \frac{\text{teachers}}{\text{students}} \quad \text{or} \quad \frac{\text{students}}{\text{teachers}} = \frac{\text{students}}{\text{teachers}}$$

$$\frac{2(\text{teachers})}{60(\text{students})} = \frac{x(\text{teachers})}{300(\text{students})}$$

$$\begin{aligned} \frac{2}{60} &= \frac{x}{300} \\ 60x &= 600 \\ x &= 10 \text{ teachers} \end{aligned}$$

$$\frac{60(\text{students})}{2(\text{teachers})} = \frac{300(\text{students})}{x(\text{teachers})}$$

$$\begin{aligned} \frac{60}{2} &= \frac{300}{x} \\ 60x &= 600 \\ x &= 10 \text{ teachers} \end{aligned}$$

Example 2: A restaurant places 8 chairs at each table. How many tables are needed to place 144 chairs.?

Solution: $\frac{\text{chairs}}{\text{tables}} = \frac{\text{chairs}}{\text{tables}}$

$$\begin{aligned} \frac{8}{1} &= \frac{144}{x} \\ 8x &= 144 \\ x &= 18 \text{ tables} \end{aligned}$$

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ACTIVITY 6 - RATIO, PROPORTION, AND PERCENT - OUTCOME 6

Explain that problems involving unit rates can be written as proportions. Emphasize that "unit" means one (1).

Example 1: Matt paid \$150 for 5 shirts. What was the cost of 1 shirt?

Solution: $\frac{\text{cost}}{\text{number of shirts}} = \frac{\text{cost}}{\text{number of shirts}}$

$$\frac{\$150}{5} = \frac{n}{1(\text{unit})}$$

$$5n = \$150 \quad (\text{Cross multiply})$$

$$n = \$30$$

Example 2: Proportions may also be used when working with scale drawings.

Using a scale of 1 cm = 2.5 m, what length line segment on a scale drawing would represent a line segment 8.75 m long?

$$\frac{\text{scale drawing}}{\text{actual segment}} = \frac{\text{scale drawing}}{\text{actual segment}}$$

Solution: $\frac{1}{2.5} = \frac{n}{8.75}$

$$2.5 n = 8.75$$

$$n = 3.5 \text{ cm}$$

MATHEMATICS
GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

OUTCOME: 10. Solve number problems for: a percent of a number, what percent one number is of another number, and the number if a percentage of the number is known.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand multiplication and division of decimals.
2. Solve simple equations.
3. Understand proportions and solve proportions with missing members.
4. Convert decimals to percents and vice versa.
5. Understand the terminology of percents.

Steps to Success

1. Help students understand that all percent statements can be viewed as a proportion.

Example: 40 is 25% of 160

The proportion involves a pair of numbers (40 and 160). One number (40) is part of the other number (160) or the whole. The percent (25) is the relationship of the part (40) to the whole (160). The statement can be viewed as follows:

40	is	25%	of	160
part	is	percent	of	whole

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Steps to Success (Cont'd)

2. Help students understand that percent problems usually take one of three forms.

- A. What is 20% of 270? Percent (20) and whole (270) are provided. Part of the whole is unknown.
B. 37.5 is what percent of 250? Part (37.5) and whole (250) are provided. Percent is unknown.
C. 100 is 20% of what number? Part (100) and percent (20) are provided. Whole is unknown.

3. Show students how to create a proportion for all percent problems. percent = $\frac{\text{part}}{\text{whole}}$

- A. Express the percent as a ratio with a denominator of 100 and write it to the left of the equals sign.

$$\frac{\quad}{100} =$$

- B. Express the pair of numbers as a ratio with the part written as the numerator and the whole written as the denominator. Write this ratio to the right of the equals sign.

$$\frac{\quad}{100} = \frac{\text{part}}{\text{whole}}$$

4. Use a variety of percent problems to show how proportions can be set up to solve them. Examples should include problems in which the percent, the part, or the whole is missing.

ACTIVITY 1 - OUTCOME 10

Give students a variety of examples similar to those below. Tell students to use proportions to solve the percent problems. Ask leading questions similar to those in the following examples until students can work problems independently.

ACTIVITY 1 - OUTCOME 10 (Cont'd)

Example 1: What is 20% of 720?

Solution: Lead a discussion similar to the following.

1. Does the problem give the percent of the number? (yes)
Therefore, write the percent as a fraction with a denominator of 100 as the left member of the proportion.

$$\frac{20}{100} =$$

2. What other term is given? Part or whole? (whole)
Remind students that the right member of the proportion is $\frac{\text{part}}{\text{whole}}$

Therefore, the right member would be

$$= \frac{N}{720}$$

3. Tell students to solve the proportion for the unknown.

$$\frac{20}{100} = \frac{N}{720}$$

$$100N = 14400 \quad (\text{cross multiply})$$

$$N = 144 \quad (\text{divide by 100})$$

Example 2: 37.5 is what percent of 250?

- Solution:
1. Is the percent of the number given? (no)
Therefore, explain that the numerator of the left member of the proportion will be the unknown.

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$$\frac{x}{100} =$$

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ACTIVITY 1 - OUTCOME 10 (Cont'd)

2. What other terms are given? (part and whole)
Remind students that the right member is $\frac{\text{part}}{\text{whole}}$

$$= \frac{37.5}{250}$$

3. Solve the proportion for the unknown.

$$\frac{x}{100} = \frac{37.5}{250}$$

$$250x = 3750$$

$$x = 15 \text{ percent or } 15\%$$

Example 3: 100 is 20% of what number?

Solution: 1. Is the percent of the problem give? (yes)
Therefore, write the percent as a fraction.

$$\frac{20}{100} =$$

2. What other term is given? Part or whole? (part)
Therefore, the right member $\frac{\text{part}}{\text{whole}}$ is written as

$$= \frac{100}{n}$$

3. Solve the proportion.

$$\frac{20}{100} = \frac{100}{n}$$

$$20n = 10,000$$

$$n = 500$$

MATHEMATICS
GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

OUTCOME: 11. Calculate discounts, simple interest, interest rates, mark-ups, tax, and other business uses of percent.

AREA: APPLICATIONS

OUTCOME: 2. Solve word problems related to consumers to include discounts, down payments, commission, taxes, ratios, and percents.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be proficient in operations with whole numbers and decimals.
2. Understand percent.
3. Change decimals to percents and percents to decimals.

Steps to Success

1. A brief review of percent is helpful.
2. Discuss business vocabulary such as discount, interest rate, mark-up, tax, percent of change, original price, gross cost, net cost, profit, sale price, marked price, time, principal, profit, assessed value, property tax, commission, down payment.

ACTIVITY 1 - RATIO, PROPORTION, AND PERCENT - OUTCOME 11
APPLICATIONS - OUTCOME 2

After discussing business vocabulary with the students, explain that many business problems have a common element. They require the computation of a percent. Business problems like the following examples usually require multiplication by the percent (rate) and addition or subtraction of the computed percent to derive the final amount asked for in the problem.

(243

244

ACTIVITY 1 - RATIO, PROPORTION, AND PERCENT - OUTCOME 11 (Cont'd)
APPLICATIONS - OUTCOME 2

Read the problem carefully. Determine the percent (rate) given in the problem, the number to be multiplied times the rate, and decide whether the problem requires another step of addition or subtraction. Addition or subtraction will often be cued by phrases or words such as reduction (-), markup (+), discount (-), and/or final cost (+ or -), depending on whether a mark-up or discount is involved.

Example 1: John Jones purchased a suit during a 25% reduction sale. Find the sale price of the suit if the marked price was \$150.

Solution: Point out the key words in this problem (25% reduction sale, sale price, marked price). Discuss the meaning of each term.

To find the sale price, compute the amount of discount and subtract that amount from the original or marked price.

1.	\$150.	marked price
	x .25	change 25% to a decimal
	750	
	300	
	\$37.50	amount of reduction

2.	\$150.00	marked price
	- 37.50	amount of reduction
	\$112.50	sale price

Emphasize the alignment of decimals before adding or subtracting.

Example 2: The Toddle Inn has a 60% profit markup on the cost of all merchandise. A baby stroller cost the merchant \$50. What will the selling price of the stroller be at Toddle Inn?

Solution: 1. The amount of profit (markup) may be found by multiplying the cost by the rate of profit (markup). \$50 (cost) x 60% (rate of profit) = amount of markup.

\$50	cost
x .60	rate of profit in decimal form
\$30.00	amount of markup

ACTIVITY 1 - RATIO, PROPORTION, AND PERCENT - OUTCOME 11 (Cont'd)
APPLICATIONS - OUTCOME 2

2. The selling price may be found by adding the cost and the amount of markup.
\$50 (cost) + \$30 (amount of markup) = \$80 (selling price)

\$50	cost
+ \$30	markup
<u>\$80</u>	selling price

Example 3: Assuming that the sales tax rate is 7%, find the total amount Mrs. Smith must pay for the stroller in example 2.

- Solution: 1. Sales tax may be found by multiplying the selling price by the sales tax rate.
\$80 (selling price) x .07 (sales tax rate decimal form) = \$5.60 (sales tax)

\$ 80	selling price
x .07	sales tax rate
<u>\$5.60</u>	sales tax

2. Total cost may be found by adding the selling price and the amount of sales tax due.
\$80 (selling price) + \$5.60 (sales tax) = \$85.60 (total cost)

\$80.00	selling price
+ 5.60	sales tax
<u>\$85.60</u>	total cost

Example 4: Mr. Jones is a car salesman who works on commission. What will Mr. Jones' weekly salary be during a week when his total sales were \$20,000 if his rate of commission is 5%.

Be sure that the students thoroughly understand commission. A brief discussion of the types of occupations in which employees are paid by the commission method would be helpful. Encourage the students to contribute to the discussion since many of their parents may work on a commission basis.

Solution: To find the salary, multiply the rate of commission by the total sales.

\$20,000	total sales
x .05	rate of commission
<u>\$ 1,000</u>	commission

ACTIVITY 1 - RATIO, PROPORTION, AND PERCENT - OUTCOME 11 (Cont'd)
APPLICATIONS - OUTCOME 2

Example 5: Mr. Smith sells insurance. His monthly salary is \$200 plus a commission of 2% of his total sales for the month. Find his total earnings for January if his January sales were \$30,000.

1. Commission: $\$30,000 \times 2\% =$
 $\$30,000 \times .02 = \$600$ (commission)
 $\$ 600$ (commission) + $\$200$ (salary) = $\$800$ (total earnings)

\$30,000	total sales
x .02	rate of commission
\$ 600	commission

\$ 200	salary
+\$ 600	commission
\$ 800	total earnings

Compare example 4 with example 5. Discuss the difference in the ways the two salesmen are paid. (In the fifth example the salesman receives a salary plus commission.) Discuss the advantages of being paid both a salary and a commission.

ACTIVITY 2 - RATIO, PROPORTION, AND PERCENT - OUTCOME 11
APPLICATIONS - OUTCOME 2

After the students become familiar with consumer problems, introduce problems with solutions that may be simplified by the use of a formula.

Example 1: Find the interest due on a \$1500 loan if the interest rate is 12% and the loan is for a three-year period.

Point out key words (interest, loan, principal, interest rate, time). Discuss the meaning of each word. Emphasize that the interest rate is usually based on the loan for one year.

ACTIVITY 2 - RATIO, PROPORTION, AND PERCENT - OUTCOME 11 (Cont'd)
APPLICATIONS - OUTCOME 2

Introduce the basic interest formula:

$$\begin{aligned} I (\text{interest}) &= P (\text{principal}) \times R (\text{rate}) \times T (\text{time}) \\ I &= \$1500 \times .12 \times 3 \\ I &= \$540 \end{aligned}$$

This problem could be expanded by asking the students to find the amount the borrower must repay at the end of 3 years. Emphasize that the borrower must repay the principal plus the interest.

$$\$1500 (\text{principal}) + \$540 (\text{interest}) = \$2040 (\text{amount that must be repaid})$$

Example 2: Mr. Jones borrowed \$1500 for three years from a local bank. At the end of three years Mr. Jones owed \$540.00 interest. Find the annual interest rate Mr. Jones paid on his loan assuming that the interest rate remained constant.

Solution: The basic formula ($I = PRT$) may be expressed in an equivalent form ($R = \frac{I}{PT}$) before solving the problem.

$$I = PRT$$

$$R = \frac{I}{PT}$$

$$R = \frac{540}{1500 \times 3}$$

$$R = \frac{540}{4500}$$

$$R = .12 \text{ or } 12\%$$

These activities may be extended to cover problems involving installment purchases with or without monthly payments, income tax, social security and other applications of percent, such as determining property tax. The procedures for computing percent in these types of problems are, for the most part, identical to the procedures covered in Activities 1 and 2.

MATHEMATICS
GRADES 7 and 8

AREA: GEOMETRY

- OUTCOMES:
1. Identify, name, and measure right, acute, and obtuse angles.
 2. Measure angles of various measure in addition to measuring right, acute, and obtuse.
 3. Identify adjacent, vertical, complementary, and supplementary angles.
 4. Identify figures and parts of figures that are congruent.
 5. Identify corresponding parts of congruent figures.
 11. State important properties of triangles and other polygons.
 12. Determine lines of symmetry.
 13. Classify triangles and other polygons according to important properties.

NOTES FOR THE TEACHER

Some of the activities for these outcomes are found also in GENERAL MATHEMATICS - INFORMAL GEOMETRY: TWO AND THREE DIMENSIONS - OUTCOME 2-6 and UNIFIED GEOMETRY - INTRODUCTION TO ANGLES - OUTCOME 3.

Prerequisites for Students

1. Identify a protractor as an instrument used to measure angles.
2. Identify the symbol for degree. ($^{\circ}$).

Steps to Success

1. The student will need to practice drawing and reading with a protractor.
2. Since so many names must be memorized in the first three activities, mnemonic devices should be utilized to assist the memorization process.

ACTIVITY 1 - OUTCOMES 1 and 2

Give each student a protractor. It is helpful to use an overhead projector when explaining the measurement of angles. Then, the teacher can use a protractor just like the students'. Have students observe the mark or arrowhead on the bottom, which will be placed on the vertex of an angle to be measured. Explain that either the

ACTIVITY 1 - OUTCOMES 1 and 2 (Cont'd)

inner scale or the outer scale can be used. The inner scale is used when one ray extends in the direction of the 0° mark on the inner scale. The outer scale is used when one ray extends in the direction of the 0° mark on the outer scale. Demonstrate. Explain that there are five kinds of angles. Draw a table like the one below illustrating names, definitions, and examples.

ANGLE NAME	DEFINITION
------------	------------

Acute	
-------	--

	contains less than 90°
--	-------------------------------

Right	
-------	--

	contains 90°
--	---------------------

Obtuse	
--------	--

	contains more than 90° but less than 180°
--	---

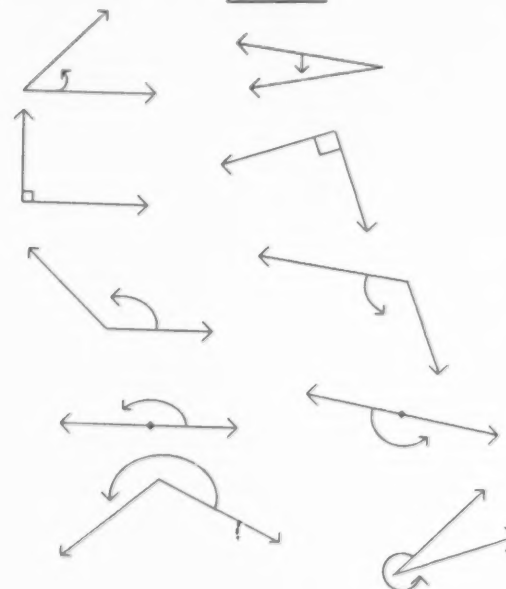
Straight	
----------	--

	contains 180°
--	----------------------

Reflex	
--------	--

	contains more than 180°
--	--------------------------------

EXAMPLE



Students can remember the names of the angles by remembering the saying "A Rose Only Smells Rosy." The first letter of each word is the first letter of each angle in order from smallest to largest.

ACTIVITY 2 - OUTCOME 3

Relate the special relationships between pairs of angles that will be used in geometry to definitions that the students already know from everyday life.

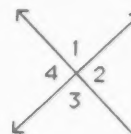
- a. Adjacent means "next to" each other. Adjacent classrooms share a wall. Adjacent neighbors in houses share a fence or property line. In condos or apartments, walls are shared. Adjacent angles share a side.



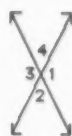
In both diagrams, $\angle 1$ is adjacent to $\angle 2$, and $\angle 1$ and $\angle 2$ share side c .

- b. Vertical means straight up and down.

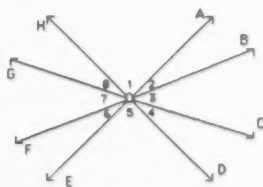
In this picture, $\angle 3$ is "straight down" from $\angle 1$. $\angle s$ 1 and 3 are said to be vertical angles.



If the picture is turned sideways, $\angle 4$ is "straight up" from $\angle 2$.



These are also vertical angles. Vertical angles are "OPPOSITE" one another, and are formed by pairs of opposite rays.



Use this diagram.

ACTIVITY 2 - OUTCOME 3 (Cont'd)

Rays $\overrightarrow{OA}$ and $\overrightarrow{OE}$ are opposite rays, $\overrightarrow{OB}$ and $\overrightarrow{OF}$ are also opposite rays; therefore, $\angle 2$ and $\angle 6$ are vertical angles. Find the other pairs of vertical angles using rays.

- c. Complementary angles are two angles whose sum equals 90° . Stress that only two angles are considered to be complements of one another. To find a complement, subtract the measure of the given angle from 90. The complement of a 35° angle is $(90 - 35) = 55^\circ$.
- d. Supplementary angles are two angles whose sum equals 180° . Again, only two angles are supplementary. Also point out that supplementary and complementary angles do not have to be adjacent, in the same picture or in the same plane. To find the supplement of an angle, subtract the measure of the given angle from 180° . The supplement of a 65° angle is $(180 - 65) = 115^\circ$.

ACTIVITY 3 - OUTCOMES 11 and 13

Explain that a polygon is a closed plane figure made up of line segments. The word polygon means many sides. The simplest polygon is a triangle because it takes at least three line segments to make a closed plane figure. The prefix in the name of the polygon tells how many sides the polygon has. The number of angles in a polygon is equal to the number of its sides.

Introduce these terms first:

Triangle- Tri means 3.

Quadrilaterals- Quad means 4.

Pentagon- Penta means 5.

Hexagon- Hex means 6.

A tricycle has 3 wheels.

Quadruplets are 4 children born in the same delivery.

The pentagon is a 5-sided building in Washington D.C.

ACTIVITY 3 - OUTCOMES 11 and 13 (Cont'd)

Next, introduce the following terms:

Septagon or Heptagon- Septa and Hepta mean 7.

Octagon- Octa means 8.

Nonagon- Nona means 9.

Decagon- Deca means 10.

Remember September.

Remember October.

Remember November.

Remember December.

Tell students that once the calendar contained only 10 months. At that time, September was the seventh month, October the eighth month, November the ninth month, and December the tenth month. Explain that some math books have "heptagon" and some say "septagon." Either word can be used for a seven sided polygon.

Students will be interested to know which two months were added to our calendar. Tell them that one was for Julius Caesar who was a Roman Emperor. Ask students which month do they think that was? (July) The other month was added for Caesar Augustus, who was really Augustus Caesar. Ask students which month do they think that was. (August)

ACTIVITY 4 - OUTCOMES 11 and 13

Since the students have already studied angles they should have no trouble classifying triangles in terms of their angles. An acute triangle is one in which all the angles are acute or measure fewer than 90° . A right triangle contains only one right angle or a 90° angle. An obtuse triangle is a triangle that contains one angle that measures more than 90° but less than 180° . Show examples of each.

Explain that triangles can be classified also by the lengths of their sides. Associate names of triangles with examples such as the following: An equilateral triangle has all sides equal or congruent. Equilateral means equal sides. An isosceles triangle has at least two equal sides. To remember this name, tell students to think of a strong man like Hercules holding something heavy over his head. In the triangle formed by his arms and the object, the length of his arms would be the same. In that triangle, the two sides formed by his arms would be equal. (Hercules sounds like- Isosceles.) On the scalene triangle no sides are equal or congruent, so a different scale is needed for each. Draw examples of each triangle. In each drawing, label sides that are equal.

ACTIVITY 5 - OUTCOMES 2 and 11



Cut out a triangle from a sheet of paper. Number each angle of the triangle. (1, 2, and 3) Cut off the three angles of the triangle and place them side by side as in the illustration. What kind of angle is formed? A straight angle is formed. Since the measure of a straight angle is 180° , the sum of the measures of the angles of a triangle is 180° .

ACTIVITY 6 - OUTCOMES 4 and 5

Congruent polygons have the same size and shape. Cut out at least three triangles two of which are congruent. Place all the triangles on the board except one of the congruent triangles which can be held in your hand. Ask, students which of the triangles on the board has the exact same size and shape as the one held in your hand. Tell students to find out by placing the hand-held triangles on top of each of those on the board. See which pair is a perfect match. The two that match are congruent polygons. Explain also that the two sides that match are corresponding parts or sides.

ACTIVITY 7 - OUTCOME 12

A figure is said to be symmetrical with respect to a line when the figure can be folded along the line and the two parts of the figure coincide. Draw several figures on the board and have students identify which figures are symmetrical with respect to a line.

Students can create ink-blot pictures which are symmetrical by placing a drop of ink on a sheet of paper and making a fold through the dot. The fold is the line of symmetry for the picture.

MATHEMATICS
GRADE 7 and 8

AREA: GEOMETRY

- OUTCOMES: 6. Construct right, acute, and obtuse angles.
7. Expand constructions to include, but not be limited to, construction of an angle equal to a given angle.
8. Construct the perpendicular bisector of a line segment.
9. Construct a line perpendicular to a given line at a point on the line.
10. Construct a line perpendicular to a given line from a point outside the line.

NOTES FOR THE TEACHER

Materials needed: wax paper

Prerequisites for Students

1. Know the definitions for and be able to identify a right angle, an acute angle, a point, a line, a line segment, a bisector, perpendicular lines, and a perpendicular bisector.
2. Be able to manipulate a compass.

Steps to Success

Most textbooks at this grade level adequately discuss constructions and the steps involved in making the constructions. Each construction is described in a simple step-by-step process. An activity that could be helpful to some students and be interesting to others would be to use wax paper to demonstrate the construction. Give students sheets of wax paper for each of the following demonstrations.

ACTIVITY 1 - OUTCOMES 6, 7, 8, 9, and 10

RIGHT ANGLES

1. Fold and crease to make a line.
2. Fold one portion of the line over the other until the lines coincide.
3. Crease.
4. Open the paper to see that the resulting lines are perpendicular; therefore, the angles are right angles

ACTIVITY 1 - OUTCOMES 6, 7, 8, 9, and 10 (Cont'd)

ACUTE ANGLES

1. Fold the paper to make a right angle as previously described.
2. Place a dot anywhere in the first quadrant.
3. Fold so that a crease is made between the dot and the vertex of the right angle.
4. Open the paper to see that the resulting angle is an acute angle.

OBTUSE ANGLES

1. Fold the paper to make a right angle.
2. Place a dot anywhere in the second quadrant.
3. Fold so that a crease is made between the dot and the vertex of the right angle.
4. Open the paper to see that the resulting angle is an obtuse angle.

PERPENDICULAR BISECTOR

1. Fold the paper and crease it to make a line.
2. Open the paper and make two dots on the crease.
3. Fold the line over itself until the line and end points coincide and make a crease.
4. Open the paper to see that the folds represent a perpendicular bisector of a line segment.

PERPENDICULAR TO A LINE AT A POINT ON THE LINE

1. Fold the paper and crease it to make a line.
2. Open the paper and make a dot on the fold.
3. Fold the line over itself until the fold is on the dot and make a crease.
4. Open the paper to see that the creases represent a line perpendicular to a given line at a point on the line.

PERPENDICULAR TO A LINE FROM A POINT OUTSIDE THE LINE

1. Fold the paper and crease it to make a line.
2. Open the paper and make a dot not on the line.
3. Fold the line over itself until the fold is on the dot and make a crease.
4. Open the paper to see that the creases represent a line perpendicular to a line from a point outside the line.

MATHEMATICS
GRADES 7 and 8

AREA: GEOMETRY

- OUTCOMES: 11. State important properties of triangles and other polygons.
13. Classify triangles and other polygons according to important properties.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand basic geometric vocabulary (ray, line, angle, segment, vertex, etc.).
2. Correctly measure an angle.
3. Classify items according to important properties.

Step to Success

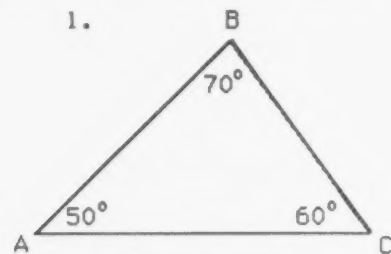
1. Emphasize the importance of learning the basic vocabulary necessary for mastering geometric concepts and for understanding geometric properties.

ACTIVITY 1 - OUTCOMES 11 and 13

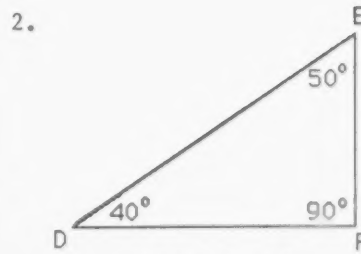
Students need to know that a triangle is a closed figure with three line segments as sides and three included angles. Triangles can be named by the measure of the angles and by the lengths of the sides. Draw figures on the chalkboard which illustrate that triangles can be classified according to the angles and lengths of the sides.

ACTIVITY 1 - OUTCOMES 11 and 13 (Cont'd)

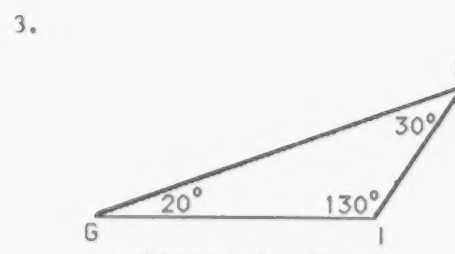
Classified According to Angles:



acute triangle
three acute angles

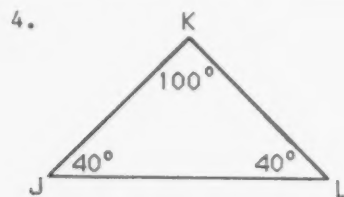


right triangle
one right angle

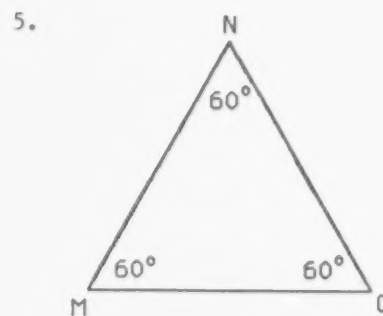


obtuse triangle
one obtuse angle

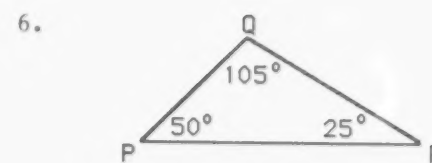
Classified According to Sides:



isosceles triangle
at least two equal sides
and two equal angles



equilateral triangle
three equal sides and
three equal angles



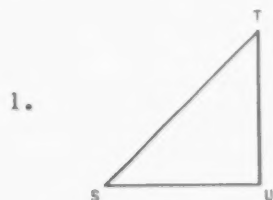
scalene triangle
three unequal sides
and three unequal angles

ACTIVITY 1 - OUTCOMES 11 and 13 (Cont'd)

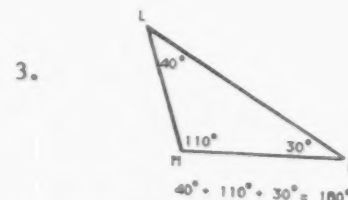
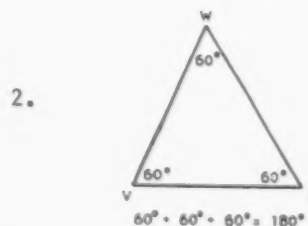
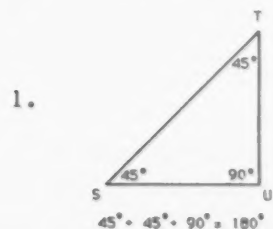
When triangles similar to the ones shown on the previous page have been drawn on the chalkboard and correctly labeled, take a sheet of paper and cover the name of a triangle. While looking at the figure whose name has been covered, have students say the name. This gives practice in associating a specific triangle with its appropriate label and practice in pronouncing its name. Use a large piece of paper to cover a triangle. Cover one at a time and avoid predictable patterns like left to right and top to bottom which would allow students to guess the triangle that would be covered next. While a given triangle is covered with the paper and only its name is visible, have the students draw a triangle at their desk that would have the characteristics of the triangle covered. (Exactness in students' drawings is not important at this point.)

ACTIVITY 2 - OUTCOMES 11 and 13

Introduce students to another important attribute of triangles: the sum of the measures of the three angles in any triangle is 180° . Provide students with drawings such as the ones found below.



It is often helpful to have students measure the degrees in several examples in order to prove that the sum of the measures of the angles is always 180°

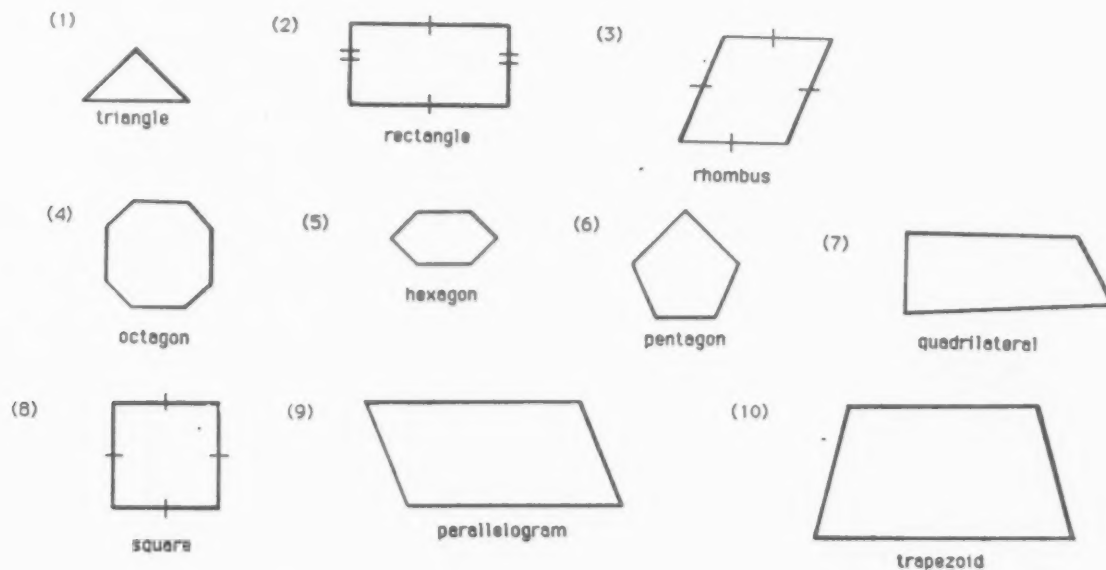


ACTIVITY 3 - OUTCOMES 11 and 13

Students need to know that polygons are closed shapes with three or more sides. Polygons are named by the number of sides they have. Introduce the vocabulary below:

1. polygon - a closed curve made up of line segments.
2. triangle - a 3-sided polygon.
3. quadrilateral - a 4-sided polygon.
4. pentagon - a 5-sided polygon.
5. hexagon - a 6-sided polygon.
6. octagon - an 8-sided polygon.
7. parallelogram - a quadrilateral with opposite sides parallel and congruent.
8. rectangle - a parallelogram with four right angles.
9. square - a rectangle with all four sides congruent.
10. trapezoid - a quadrilateral with only one pair of opposite sides parallel.
11. rhombus - a parallelogram with all four sides congruent.

Illustrate each of the polygons on the chalkboard and label them. The board might look similar to the drawing below.

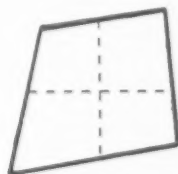


ACTIVITY 3 - OUTCOMES 11 and 13 (Cont'd)

As in Activity 1, the teacher can cover with a sheet of paper some of the names of the polygons. While the label is covered, have students pronounce the name and then write it on a sheet of paper. When all students have attempted to spell the name of the polygon correctly, the teacher can uncover the name. Students then check their spelling for correctness.

ACTIVITY 4 - OUTCOMES 11 and 13

Polygons may be classified as convex or concave. A polygon is convex if any segment connecting two points on the polygon lies wholly within the polygon. Otherwise, the polygon is concave.



convex



concave

Students who have difficulty distinguishing between convex and concave polygons often benefit from using models as illustrations. Take a model of a polygon and stretch a rubber band around the model. If any part of the rubber band does not lie on a side of the polygon, the polygon is concave. Otherwise, it is convex.

MATHEMATICS
GRADES 7 and 8

AREA: GRAPHS, MAPS, AND CHARTS

OUTCOMES: 1. Read and plot number pairs on a grid.

NOTES FOR THE TEACHER

The activity for this outcome is found also in ALGEBRA I - FIRST DEGREE EQUATIONS AND INEQUALITIES CONTAINING TWO VARIABLES - OUTCOMES 1 and 2.

Prerequisites for Students

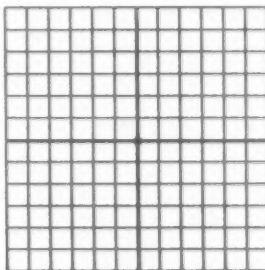
1. Recognize and understand signed numbers.
2. Graph numbers on a number line.

Step to Success

1. The students should transfer the knowledge of straight line plotting to rectangular grid plotting.

ACTIVITY 1 - OUTCOME 1

The following is a diagram of a city laid out in perfect square blocks. The address of each intersection is given by how far to the left or right and how far up or down one must travel from the center intersection, which is called the origin, and can be represented by the ordered pair $(0,0)$.

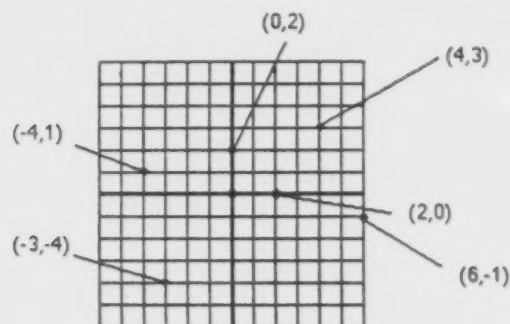


ACTIVITY 1 - OUTCOME 1 (Cont'd)

The first number in the ordered pair is how far they have traveled to the left or right, and the second number is how far they have traveled up or down. Travel to the right is a positive direction (+) and to the left is a negative direction (-). Similarly, up is positive (+), and down is negative (-).

Find the addresses for $(6, -1)$, $(-4, 1)$, $(2, 0)$, $(0, 2)$, $(-3, -4)$, and $(4, 3)$.

Place a dot on the intersection for each ordered pair.



MATHEMATICS
GRADES 7 and 8

AREA: GRAPHS, MAPS, AND CHARTS

- OUTCOMES: 2. Collect data and construct charts and graphs.
3. Interpret and use data from graphs, tables, charts, diagrams, maps, and scale drawings to solve problems.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Add, subtract, multiply and divide whole numbers, fractions, and decimals.
2. Recognize frequency distribution tables and other methods of collecting data.

Steps to Success

Emphasis should be placed on the value of reading and interpreting information shown in graphs, tables, charts, diagrams, maps, and scale drawings. Use statements like "a picture is worth a thousand words." Graphs show information concisely that would otherwise take pages to show in words. Graphs, tables, charts, diagrams, maps, and scale drawings are useful in presentations. The audience will usually be more attentive if the information is presented in an orderly graph or table. Scale drawings save space and are useful in industry, as well as in our private lives. Maps drawn to scale are useful to consumers.

ACTIVITY 1 - OUTCOMES 2 and 3

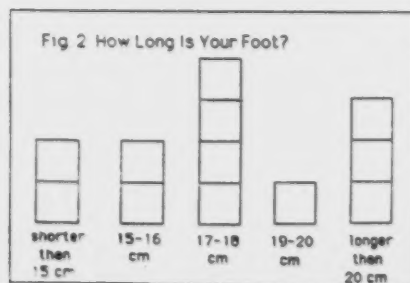
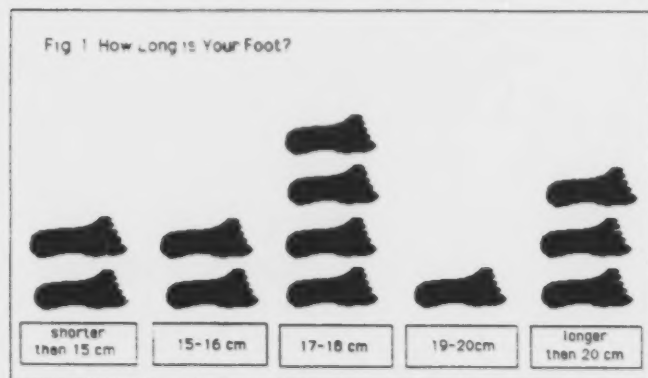
Introduce students to the four types of graphs. Two of these, the picto-graph and the bar graph, are similar. They show different amounts by using pictures or bars. The pictures will be labeled to show what they represent. The number that goes with each picture can be read to interpret the facts given in the graph. Bars of different lengths are used on a bar graph, and their value can be determined by where the bar ends on the graph.

ACTIVITY 1 - OUTCOMES 2 and 3 (Cont'd)

The circle graph looks like a pie and is divided into wedges. The circle represents the whole thing that is described. It is divided into wedges that stand for the different parts of the whole. Look for the number that represents each wedge or part.

A line graph is the fourth graph students need to be able to read. A line graph is often used to show how something has changed over a period of time. Information is given at the left side of the graph and across the bottom. The number that represents the information needed can be determined by finding where the information given at the left crosses the information given at the bottom.

Have Students make graphs to represent data and to help make the data understandable quickly. Direct students in making a graph similar to the following.



ACTIVITY 1 - OUTCOMES 2 and 3 (Cont'd)

Have children use plain paper to make careful outlines of their feet. Then, have them cut out their footprints, measure them with a ruler, label them, and pin them on a bulletin board in categories to show their approximate sizes.

After students have made the foot graph, they can also represent the data in more abstract formats using traditional small-sized graphs (Fig. 2). The class might use purchased graph paper or centimeter-squared paper provided by the teacher. For the foot graph, the class can represent each student's foot by a square on their paper. Teachers should lead the class in comparing the life-sized and scaled-down graphs.

Provide other examples of graphs and ask students questions relating to the information contained in them.

Example: Look at the circle graph of the world's oceans and answer the questions below.



1. Which is the largest ocean? (Pacific)
2. Which is the smallest ocean? (Arctic)

ACTIVITY 2 - OUTCOMES 2 and 3

Provide students with tables and charts that contain large amounts of information. Tables and charts like these will enable the teacher to ask numerous questions about one chart or one table. Choose examples similar to the one that follows.

ACTIVITY 2 - OUTCOMES 2 and 3 (Cont'd)

Example 1: Read the table below and answer the questions that follow.

Average Weight of Sports Equipment

Sport	Weight (Ounces)	Sport	Weight (ounces)
Baseball	5.25	Rugby Ball	14.5
Basketball	22.045	Soccer Ball	15.1
Croquet Ball	16.05	Softball	6.625
Football	14.8	Squash Ball	0.868
Golf Ball	1.62	Tennis Ball	2.03
Table Tennis Ball	0.088	Volleyball	9.525

1. Which is lighter, a football or a croquet ball?
(football)
2. Order the weights of the following from the greatest to the least:
baseball, squash ball, softball, tennis ball. Write the name and
the weight of each.
(softball - 6.625; baseball - 5.25; tennis ball - 2.03; squash ball - .868)
3. What would be the heavier weight, 10 volleyballs or 100 golf balls?
(100 golf balls)

ACTIVITY 3 - OUTCOME 3

Problems that involve scale drawings can be solved by setting up proportions. Before providing activities related to scale drawings, teachers should be certain that students can set up proportions and solve them. (See Ratio, Proportions, and Percents, Outcomes 1-6.)

ACTIVITY 3 - OUTCOME 3 (Cont'd)

In all problems involving scale drawings, three terms of a proportion are provided.

Example: The scale on a map of Tampa is 1 cm = 6 miles. The distance from Crater Park to Oak Park is represented by 4 cm. How far is it in miles between Crater Park and Oak Park?

The three known terms are 1 cm, 6 miles, and 4 cm.

In addition, problems involving scale drawings establish one relationship. In the example above

$$1 \text{ cm} = 6 \text{ miles}$$

All problems involving scale drawings ask for a second relationship. In the example above the question is:

$$4 \text{ cm} = ? \text{ miles}$$

Instruct students to use a proportion to find the unknown term. In setting up the proportion, students will benefit from stating the proportion using words only. In the example above, the proportion would be:

1 cm is to 6 miles as 4 cm is to ____ miles.

Through this proportion, students should be able to set up the proportion and solve it

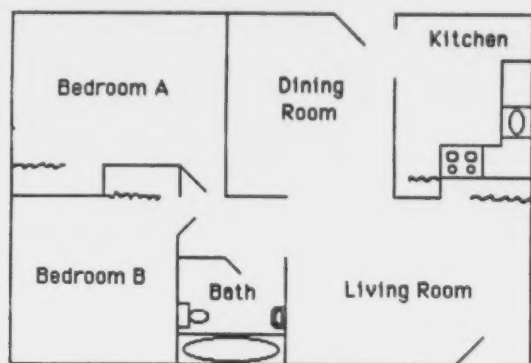
$$\frac{1}{6} = \frac{4}{x}$$

$$x = 24 \text{ miles}$$

Provide problems similar to the following, that include a scale drawing and some scale representation.

ACTIVITY 3 - OUTCOME 3 (Cont'd)

Example: Use the scale drawing below to determine the width of the apartment.



Scale: 1 inch to 12 feet

Solution: Relationship given: 1 inch = 12 feet.

Relationship asked for: $2\frac{3}{4}$ inches = ____ feet.

Set up proportion in words: 1 inch is to 12 feet as $2\frac{3}{4}$ inches is to ____ feet.

Set up mathematical representation: $\frac{1}{12} = \frac{2\frac{3}{4}}{x}$

Solve:

$$x = 12\left(2\frac{3}{4}\right)$$

$$x = 12\left(\frac{11}{4}\right)$$

$$x = 33 \text{ feet}$$

MATHEMATICS
GRADES 7 and 8

AREA: STATISTICS AND PROBABILITY

- OUTCOMES:
1. Compute the range, mean, median, and mode of a group of numbers.
 7. Demonstrate knowledge of sampling.
 8. Use a sample to predict an average property of a population.
 9. Use a sample to determine probabilities.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Be able to add, subtract, multiply, and divide.
2. Understand descending and ascending order.

Steps to Success

Students must understand the meaning of the following terms:

- a. data - The information to be analyzed. (It may be a set of test scores, ages, hair color, heights, etc.)
- b. range - The difference between the smallest and the largest numbers in the set of data.
- c. mean - An average. (It may be found by adding the data and dividing by the number of data.)
- d. median - The middle number in a set of data when the data is arranged in ascending or descending order is the median. In cases where the number of data is even, average the two middle numbers to obtain the median.
- e. mode - The number (or numbers) that occurs most often in a set of data. (A set of data may have no mode-no number occurs more often, one mode-a single number occurs more often, or several modes-more than one number occurs the same number of times.)

ACTIVITY 1 - OUTCOME 1

Determine the range, mean, median, and mode of data by using the definitions of each term given in the Steps to Success and the information given in each example. To first order all information (data) in either ascending or descending order is an additional step that is often helpful.

Example 1: Data: 8, 15, 12, 15, 5

Find the range, mean, median, and mode.

Solution: Arrange the data in ascending or descending order.

Ascending order: 5, 8, 12, 15, 15

Range: largest number 15
 - smallest number - 5
 range 10

Mean: $5 + 8 + 12 + 15 + 15 = 55$
 55 divided by 5 = 11
 11 is the mean.

Median: Since the data contains an odd number of items, the middle number (12) is the median.

Mode: The number 15 occurs twice. Since no other number occurs two or more times, the mode is 15.

ACTIVITY 1 - OUTCOME 1 (Cont'd)

Example 2: Data: 20, 30, 40, 50

Find the range, mean, median, and mode.

Solution: The data is already listed in ascending order.

Range: largest number 50
 - smallest number - 20
 range 30

Mean: 20 + 30 + 40 + 50 = 140
 140 divided by 4 = 35
 mean = 35

Median: 20, 30, 40, 50

Since the data contains an even number of items, two numbers (30, 40) are "in the middle."

To find the median, find the mean of the two middle numbers.

Add 30 + 40 = 70
Divide 70 by 2 = 35
The median is 35

Mode: This set of data has no mode since no number occurs more often than another number.

Remind the students not to list the mode of this data as 0. 0 is a number. They should say there is no mode.

ACTIVITY 2 - OUTCOMES 1, 7, 8, and 9

Ask students to count the number of coins in their pockets or purses. Record the various numbers of coins on the board using a table such as the following:

No coins	//	2	3 coins	///	5	Over 5 coins	///	//	7
1 coin	/	1	4 coins	///	3				
2 coins	///	3	5 coins	////	4				

Ask students to:

- A. find the range $7 - 1 = 6$
- B. find the mean $\frac{2 + 1 + 3 + 5 + 3 + 4 + 7}{7} = \frac{25}{7} = 3.57$
- C. find the mode over 5 coins
- D. find the median 4 coins

Have the students to write their predictions for each number of coins the class will have for the next day. determine the range, mean, mode, and median each day. Which measures vary the most? Continue this activity for several days. Let the students determine if their predictions are becoming more accurate.

ACTIVITY 3 - OUTCOMES 7, 8, and 9

Poll the class. Ask the student to tell the number of brothers and sisters they have. Compile the data. Have the students compute the range, mean, median, and mode for the number of brothers, the number of sisters, and the number of brothers and sisters.

Ask the class which of these statistical items is or is not representative of the data. (Are any of the items in the set of data extremely high or extremely low?) Sometimes items that are extremely high or low influence the outcome of the statistics.

ACTIVITY 3 - OUTCOMES 7, 8, and 9 (Cont'd)

How do these extremely high or low items influence the range, mean, median, and mode? Which measures are influenced the most? Which measures are influenced the least?

Have the students make predictions about other classes based on the information they have found to exist for their class. Poll the other classes. Compute the range, mean, median, and mode for their information, and determine if the students' predictions were accurate.

MATHEMATICS
GRADES 7 and 8

AREA: STATISTICS AND PROBABILITY

- OUTCOMES:
2. Demonstrate knowledge of the use of statistics.
 3. Demonstrate knowledge of vocabulary of probability.
 4. Count or determine possible outcomes of an event.
 5. Compute the probability of an event.
 6. Determine the odds of an event.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Understand ratios.
2. Have mastery of basic operations with whole numbers, decimal numbers, and fractions.
3. Compute averages.

Steps to Success

1. The key to understanding the probability of an event is the students' ability to recognize the possible outcomes and to carry out simple experiments to determine probability.
2. Range, mean, median, and mode must be defined.
3. Before using a graph or a table to organize data, the data must first be ordered.

ACTIVITY 1 - OUTCOMES 2, 3, 4, 5, and 6

Ask students if they have heard of people who predict the future. These people are sometimes called psychics. Explain that there are other people who need to predict events. For example, the owner of a store must be able to predict (or estimate) how many of a certain item he can sell when he orders his goods. How does he make his prediction? Point out that the store owner bases his prediction on past events. Explain that the probability of future events can be predicted by experimenting. The possible outcome must be known before the probability of an event occurring can be predicted.

For example, if a coin is flipped, how many different outcomes are possible?

There are only two possible outcomes. Either the coin will land showing "heads" or "tails."

Divide the class into four groups. Each group will have each student flip a coin 3 times. One member of each group will record the number of "heads" and "tails" shown. Totals will be added at the board.

The experiment shows that two results were recorded: "heads" and "tails." Introduce students to the probability formula:

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Explain that if the desired outcome is heads, the probability is $1/2$ because there is 1 favorable outcome (heads) and 2 possible outcomes (heads and tails).

Repeat this type of experiment using a die. Ask if anyone can predict the probability of rolling a three on a single die? The probability is $1/6$ since there is one favorable outcomes and six possible outcomes. Ask what the probability is of tossing a seven? Since there is no seven on a die, the number of favorable outcomes is zero. Therefore, the probability of tossing a seven is 0 ($0/6 = 0$).

ACTIVITY 2 - OUTCOMES 2, 3, 4, 5, and 6

ACTIVITY 1 can be expanded to include problems with single and multiple outcomes. Review the formula used in ACTIVITY 1.

$$\text{Probability (P)} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Point out that in single outcome problems, only one favorable outcome is desired.

Example 1: What is the probability of tossing 5 on a die?

Solution 1: $P = 1/6$

Explain that in multiple outcome problems, more than one favorable outcome is named.

Example 2: What is the probability of tossing either a 2 or 3 on a die?

Solution 2: $P(3 \text{ or } 2) = P(3) + P(2)$

$$P(3 \text{ or } 2) = 1/6 + 1/6$$

$$P(3 \text{ or } 2) = 2/6 \text{ or } 1/3$$

Work with students to make up additional multiple-outcome problems.

ACTIVITY 3 - OUTCOMES 2, 3, 4, 5, and 6

The probability of independent events occurring can be calculated using simple probability formulas. Start by asking how students could determine the probability of flipping a coin with "heads" and rolling a die to show 3. Direct students to look at all of the possible outcomes of flipping a coin and rolling a die. Let "H" = heads and "T" = tails. Let the numerals 1-6 stand for what can be shown when a die is rolled. List the possible combinations for the coin flip and the die toss.

ACTIVITY 3 - OUTCOMES 2, 3, 4, 5, and 6 (Cont'd)

Example: H1 T1
 H2 T2
 H3 T3
 H4 T4
 H5 T5
 H6 T6

There are 12 possible outcomes but only 1 favorable outcome. Therefore, the probability of tossing a heads and rolling a 3 is $1/12$.

This solution helps to show that the formula for the probability of independent events occurring is valid. The formula is $P(A \text{ and } B) = P(A) \times P(B)$.

Determine the probability of dependent events varies slightly. For dependent events, the first event has some effect on the probability of the second event.

Example: Suppose a bag contains 10 marbles: 7 red, 2 white, and 1 black. What is the probability of picking a red marble on the first draw and a white marble on the second draw? (The first draw is not put back in the bag.)

Solution: The probability of picking a red marble first is equal to $\frac{\text{favorable outcomes}}{\text{total possible outcomes}}$ or $\frac{7}{10}$.

Do not replace the marble that was picked. Ask students if the probability of drawing a white marble on the next pick is equal to $2/10$? Explain that the answer is no, because there are only 9 marbles left in the bag. Therefore, the probability of choosing a white marble on the second pick is $2/9$.

The probability of picking a red marble on the first draw and a white marble on the second draw is

$$\frac{7}{10} \times \frac{2}{9} = \frac{14}{90} \text{ or } \frac{7}{45} .$$

MATHEMATICS
GRADES 7 and 8

AREA: STATISTICS AND PROBABILITY

- OUTCOMES:
10. Gather data to construct a sample.
 11. Analyze data taken for a sample.
 12. Report data using a histogram.
 13. Read a frequency distribution.
 14. Make and read a frequency distribution chart.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have mastery of basic operations.
2. Understand ascending and descending order.
3. Understand rounding.
4. Understand midpoint and range.

Steps to Success

Emphasis should be placed on students' being precise in their counting and recording. It is often helpful if a straight edge is used to assist in reading information from distribution or chart.

ACTIVITY 1 - OUTCOMES 10, 11, 12, 13, and 14

A list of information or a group of measurements is of little value until it has been summarized, analyzed, and then interpreted. Once the information or measurements have been organized and a summary made, the many properties and much information can be derived from the original set of information or measurements.

Write the test scores of the class for the last two or three major tests on the board. (Do not list names, just the numerical scores.) Lead the class through the gathering and ordering of this information to recording it in a frequency distribution, histogram, and then analyzing some of the data contained in the frequency distribution or histogram.

ACTIVITY 1 - OUTCOMES 10, 11, 12, 13, and 14 (Cont'd)

Example 1: Use the ordered list of test scores below to develop a frequency distribution.

49	59	63	67	70	73	75	77	82	87
52	60	63	68	71	73	75	79	84	89
56	60	65	69	71	73	75	79	84	92
58	62	66	69	72	74	76	80	85	93
59	63	67	70	73	74	76	80	87	97

- Solution:
1. The number of intervals used in the distribution is usually a matter of choice, but remember that too many intervals will be hard to manage and too few will construe the information. A good rule of thumb is that 10-20 intervals are the most practical. Since there are fifty scores, choose an interval of 10.
 2. Determine the range of scores (largest score 97 minus the lowest score 49). The range is 48 and the number of intervals selected was 10; therefore, the interval size would be $\frac{48}{10}$ or 5 rounded to the nearest integer.
 3. The top interval should include the highest score and the lowest interval should include the lowest score. When selecting the top interval, also have its midpoint divisible by the interval size.

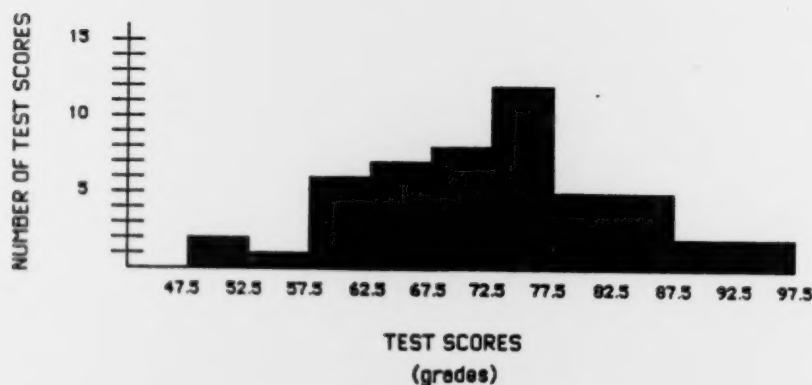
The frequency would look like the one below.

<u>Scores</u>	<u>Midpoint</u>	<u>Tabulation</u>	<u>Frequency</u>
93-97	95	//	2
88-92	90	//	2
83-87	85	++++	5
78-82	80	++++	5
73-77	75	++++ +++++ //	12
68-72	70	++++ +++++	8
63-67	65	++++ //	7
58-62	60	++++ /	6
53-57	55	/	1
48-52	50	//	2

ACTIVITY 1 - OUTCOMES 10, 11, 12, 13, and 14 (Cont'd)

Example 2: Use the frequency distribution in Example 1 to represent the data graphically in a frequency histogram.

Solution: A frequency histogram is similar to a bar graph. It has a rectangular base equal to the interval size and a height equal to the number of cases in that interval. Since a test score of 82 represents a score between 81.5 and 82.5, the intervals will, for example, include scores between 47.5 and 52.5 for the lowest interval.



Example 3: Have students answer questions similar to the following to demonstrate their understanding of the information contained in the frequency distribution and in the histogram.

- What is the midpoint of the scores 63-67?
- How many students had scores between 58-62?
- Which interval had the most scores? the least?
- Which interval had only one score fall within its limits?

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 1. Use knowledge of conversion facts to change units of time to solve problems.
2. Add and subtract units of time and convert among the units.
7. Solve problems related to time allocation.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Should be able to tell time.
2. Know know the number of minutes in an hour, the number of hours in a day, the number of days in a week, and the number of months in a year.

Steps to Success

1. Students will need practice making conversions between the following times:

minutes to hours	hours to minutes
hours to days	days to hours
days to weeks	weeks to days
months to years	years to months

2. The student must know:

60 minutes	= 1 hour
24 hours	= 1 day
7 days	= 1 week
12 months	= 1 year

Steps to Success (Cont'd)

3. In making conversions involving time, the first step in every problem is to decide if the conversion is from a smaller unit to a larger unit (like minutes to hours, hours to days) or from a larger unit to a smaller unit (like hours to minutes, days to hours). If the conversion is from a smaller unit to a larger one, students must always divide. It takes fewer of a larger unit to equal a given number of smaller units. If the conversion is from larger units to smaller ones, students must always multiply. It takes more of a smaller unit to equal a given number of larger units.

ACTIVITY 1 - OUTCOMES 1, 2, and 7

Example 1: From Smaller Units to Larger Units

During one week Jan jogged 30 minutes, 25 minutes, 45 minutes, and 20 minutes. How many hours did she jog?

Solution: (1) Decide what unit of measure is given. (minutes) Find the total minutes jogged. (120 minutes)

(2) Look for the unit of measure to which you are changing. (hours)

(3) Determine how many minutes are in one hour. (60)

Divide 60 into 120, the unit of measure given. ($120 \div 60 = 2$)

(4) The answer 2 is the number of hours in 120 minutes.

Example 2: From Larger Units to Smaller Units

Matt worked every day for 3 weeks during the summer vacation. How many days did Matt work?

ACTIVITY 1 - OUTCOMES 1, 2, and 7 (Cont'd)

Solution: (1) Find the unit of measure given. (3 weeks)

(2) Look for the unit of measure to which you are changing. (days)

(3) Decide how many days are in one week. (7)

Multiply 7 times (3), the unit of measure given. ($7 \times 3 = 21$)

(4) The answer is 21 days. Or, 21 days are equivalent to three weeks.

Have students develop word problems which require them to change units of time. These can be exchanged in class or compiled into a class test.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 3. Determine beginning, ending, and elapsed time.
4. Solve problems involving time by determining beginning, ending, and elapsed time.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Tell time.
2. Add and subtract.
3. Convert time to equivalent expressions, so that subtraction is possible.

Steps to Success

1. Inform students that each problem will contain two times. Students will be asked to find a starting time, a finishing time, or elapsed time.
2. The first activity will use drawings of clocks to illustrate the meaning of elapsed time. When the meaning has been demonstrated, students will be shown that a quick solution to finding elapsed time is through subtraction.
3. Activity 2 will give examples which show that determining beginning time requires subtraction but determining ending time requires addition.

ACTIVITY 1 - OUTCOMES 3 and 4

One way to approach problems involving elapsed time is to think of the clock as being divided into four 15-minute periods. Remind students that each time the minute hand makes a complete circle around the clock, one hour has passed.

ACTIVITY 1 - OUTCOMES 3 and 4 (Cont'd)

Example: Determine the number of hours between 6:45 p.m. and 9:15 p.m.

Solution: (1) Start at 45 minutes. THINK 6:45 to 7:45 = 1 hour. Make one complete circle around the clock (from 45 to 45). Make a tally mark representing one hour. /



(2) THINK 7:45 to 8:45 = 1 hour. Make another complete circle around the clock. Make another tally mark representing another hour. //



(4) Now there are 2 tally marks representing 6:45 to 8:45 or 2 hours.

(5) Use the circle clock to find the time from 8:45 to 9:15. It involves two 15-minute periods.



(6) Therefore, the two tally marks mean two hours and the two 15-minute periods equal thirty minutes or a half hour.

The answer is 2 hours and 30 minutes or 2 1/2 hours.

A quicker solution to problems involving elapsed time, however, is to find the difference between the starting time and the ending time or to subtract.

Example: Determine the number of hours between 6:45 p.m. and 9:15 p.m.

Solution: 9:15 - Ending time
 -6:45 - Beginning time
 - Elapsed time

ACTIVITY 1 - OUTCOMES 3 and 4 (Cont'd)

Rewrite the ending time and subtract.

$$\begin{array}{r} 9:15 = 8 \text{ hours and } 75 \text{ min.} \\ - 6:45 = 6 \text{ hours and } 45 \text{ min.} \\ \hline 2 \text{ hours and } 30 \text{ min. or } 2 \frac{1}{2} \text{ hours} \end{array}$$

ACTIVITY 2 - OUTCOMES 3 and 4

Point out that determining the ending time requires that the elapsed time be added to the beginning time.

Example: The football game started at 7:30 and lasted 2 1/2 hours. At what time did the game end?

Solution: $\begin{array}{r} 7:30 \\ + 2 \frac{1}{2} \\ \hline \end{array}$ Beginning time
Elapsed time
Ending time

Emphasize the importance of rewriting problems to contain the same units of measure.

$$\begin{array}{r} 7:30 = 7 \text{ hours } 30 \text{ minutes} \\ + 2 \frac{1}{2} \text{ hr.} = + 2 \text{ hours } 30 \text{ minutes} \\ \hline 9 \text{ hours } 60 \text{ minutes} \end{array}$$

Emphasize the importance of expressing the answer using the terminology common to telling time.

9 hours 60 minutes = 10 o'clock.

To provide needed practice, have students write word problems which ask for the beginning time, the elapsed time, and the ending time. Instruct students to include at least one example of each type of problem in their work.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES:**
5. Determine elapsed time between two different zones.
 6. Determine the amount of time allocated for an activity.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Mastery of basic operations with whole numbers, decimal numbers, and fractions.
2. Addition and subtraction with hours, minutes, and seconds.

Steps to Success

1. Emphasize to students that elapsed time involves a beginning time and ending time, and/or a duration (elapsed time). Two of these terms are provided in the problem and the third term must be calculated either by addition or subtraction.
2. Units of time should be simplified if possible.
3. Students need to know the different time spaces and the time changes between different zones.

ACTIVITY 1 - OUTCOME 5

Elapsed time involves locating the beginning time and ending time and subtracting to find their difference. When beginning or ending time is given in different time zones convert all times to one zone and then subtract.

Example: A football game started at 7:30 p.m. EST and ended at 8:45 p.m. CST. How long did the game last?

Solution: Convert all time to either EST or CST.
7:30 p.m. EST is the same as 6:30 p.m. CST.

$$\begin{array}{r} 8:45 \text{ p.m. CST} \\ - 6:30 \text{ p.m. CST} \\ \hline 2:15 \end{array}$$

2:15 or two hours fifteen minutes is the elapsed time.

330

331

ACTIVITY 2 - OUTCOME 6

Review adding, subtracting, multiplying and dividing units of measure, if necessary. Review converting units of measure hours to minutes to seconds and vice versa, if necessary.

Have students solve practical problems involving time.

Example: A coach wanted to spend $\frac{2}{3}$ of practice time on one drill. How much time can be spent on the drill if practice lasts for 1 hr. 45 min.?

Solution: 1 hr. 45 min. = $1 \frac{45}{60} = 1 \frac{3}{4}$ hr.

$$\frac{2}{3} \text{ of } 1 \frac{3}{4} \text{ hr.} = \frac{2}{3} \times 1 \frac{3}{4} =$$

$$\frac{2}{3} \times \frac{7}{4} =$$

$$\frac{14}{12} =$$

$1 \frac{1}{6}$ hr. or 1 hr. 10 min. since $\frac{1}{6}$ of 60 minutes is 10 minutes.

or

Since 1 hr. 45 min. = 105 min.

$$\frac{2}{3} \text{ of } 105 = \frac{2}{3} \times \frac{105}{1}$$

$$= \frac{210}{3}$$

= 70 min. or 1 hr. 10 min.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

OUTCOME: 9. Measure lengths using customary and metric units.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have access to metric and customary rulers.
2. Recognize whole number and fractions.
3. Compare whole numbers and fractions.
4. Add whole numbers, decimals and fractions.
5. Recognize equivalent fractions.

Steps to Success

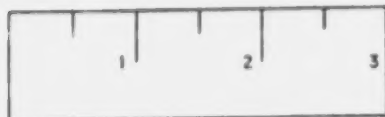
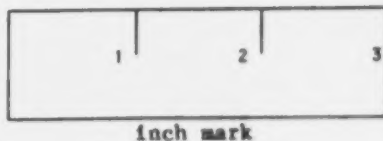
1. Give examples of practical applications of this exercise, such as in shop classes, home economic classes, etc., where accurate measurement is essential.
2. Allow student time to measure random items in the classroom with measuring devices, such as a one-foot ruler.
3. Illustrations in the examples to follow are not drawn to scale. Larger drawings on the chalkboard may help during instruction. All examples provided students, however, should be drawn to scale.

ACTIVITY 1 - OUTCOME 9

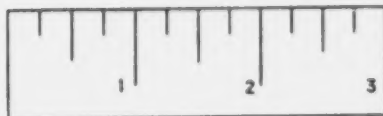
CUSTOMARY MEASURE

The key to accurate measurements is to recognize that the inch marks on the customary ruler are the longest marks. They are labeled 1, 2, 3, 4, 5, 6, etc. The marks between the inch marks become successively shorter beginning with $\frac{1}{2}$ and continuing to get shorter by each division: $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$.

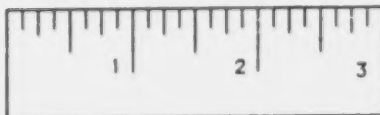
ACTIVITY 1 - OUTCOME 9 (Cont'd)



Note that half-inch mark is shorter than inch mark.



Note that fourth-inch mark is shorter than half-inch mark.



Note that eighth-inch marks are shorter than fourth-inch marks.



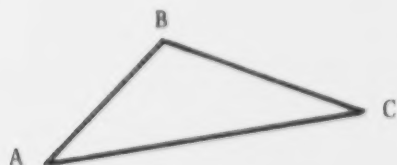
Note that sixteenth-inch marks are shorter than eighth-inch marks.

ACTIVITY 1 - OUTCOME 9 (Cont'd)

Being familiar with equivalent fractions can help in reading measurements. Drawings on the chalkboard can remind students that $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ are equivalent names for the fraction $\frac{1}{2}$. Likewise, $\frac{2}{8}$ and $\frac{4}{16}$ are equivalent to $\frac{1}{4}$.

To measure the length of a line, place the beginning of the ruler at the beginning of the line. Read the number on the ruler where the line segment that you are measuring ends. This is the length of the line.

Example: Find the length in inches from A to C by way of B.



Solution: (1) Measure the length of line segment $\overline{AB}$. 1 inch

(2) Measure the length of line segment $\overline{BC}$. $1 \frac{1}{4}$ inch

(3) Add these measures together to find the distance from A to C by way of B. $2 \frac{1}{4}$ inches

ACTIVITY 2 - OUTCOME 9

METRIC MEASURE

With practice, the metric side of the ruler is even easier to read than the standard ruler. The metric ruler is divided into only two measures. The longer lines represent centimeters and are numbered 1, 2, 3, 4, etc. There are 10 shorter lines between centimeters that are called millimeters.

ACTIVITY 2 - OUTCOME 9 (Cont'd)



Place the beginning of the ruler at the beginning of the line. Read the measures on the ruler where the line segment ends. This is the length of the line.

Example: Find the length in centimeters of the line segment from A to B.

A _____ B

Solution: Place the beginning of the ruler at the beginning of the line. Read the measure on the ruler where the line segment ends. This is the length of the line. (5 centimeters)

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

OUTCOMES: 10. Measure capacities using customary and metric units.
11. Solve problems involving capacities.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Follow directions.
2. Add decimals, fractions, and whole numbers.

Steps to Success

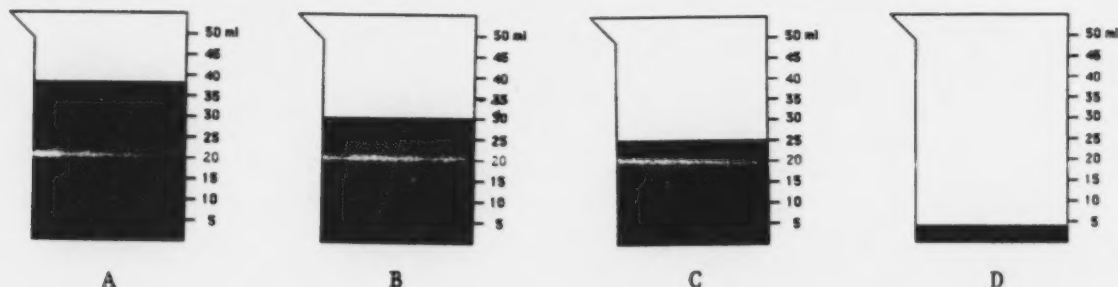
1. Students are more successful with this activity if visual aids are used.
2. Students need to use both standard and metric containers.

ACTIVITY 1 - OUTCOMES 10 and 11

Capacity measurement requires students to solve one of the following two types of problems. The first type requires students to look at different pictures and decide which one represents the amount of liquid stated in the problem. To solve this problem, students observe which numbers are given at the side of each drawing and decide in which drawing the amount needed is shaded.

ACTIVITY 1 - OUTCOMES 10 and 11 (Cont'd)

Example 1: Which container has exactly 25 milliliters of liquid?



Solution: (a) The shaded part of A is about 38 milliliters.
(b) The shaded part of B is 30 milliliters.
(c) The shaded part of C is 25 milliliters.
(d) The shaded part of D is about 4 milliliters.

Therefore, C is the correct answer.

Note that each answer indicates the correct quantity in its picture. Choice C is correct because only C answers the Example question.

The second type of problem provides measures for two or more containers. Students are asked to find the total amount of liquid.

Example 2: A $6 \frac{1}{2}$ ounce can of soup is to be mixed with $6 \frac{1}{2}$ ounces of water. How many ounces of liquid will there be?

Solution: There are two measures given in the problem. They must be added together to find the total.

$$\begin{array}{r} 6 \frac{1}{2} \text{ ounces of soup} \\ + 6 \frac{1}{2} \text{ ounces of water} \\ \hline 12 \frac{2}{2} = 13 \text{ ounces of liquid} \end{array}$$

In providing examples for student practice, be sure to include dry measures (bushels, pecks, ounces, pounds, etc.), as well as liquid measures.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

OUTCOMES: 15. Add and subtract with customary measures.
16. Compute with customary units.

NOTES FOR THE TEACHER

The activities for these outcomes are found also in GENERAL MATHEMATICS - MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS - OUTCOMES 15, 21, 27, 28, and 34.

Prerequisite for Students

1. Perform the four basic operations with whole numbers, fractions, and decimals.

Steps to Success

1. All like units of measure should be kept in the same column when adding or subtracting.
2. Units of measure should be simplified if possible.

ACTIVITY 1 - OUTCOMES 15 and 16

Ask students if they remember how their elementary school teacher taught them to add. Demonstrate with an example similar to the following:

$$\begin{array}{r} 29 \\ +48 \\ \hline \end{array}$$

First add the units $\begin{array}{r} 9 \\ + 8 \\ \hline 17 \end{array}$

$$\begin{array}{r} 1 \\ 29 \\ +48 \\ \hline 7 \end{array}$$

17 is more than 1 ten, so write down 7 and carry the 1 ten. (Regroup)

$$\begin{array}{r} 1 \\ 29 \\ +48 \\ \hline 77 \end{array}$$

Now add the 1 ten to the other numbers in the tens column.

77 is the sum.

Adding units of measure involves a similar procedure.

Example 1:

$$\begin{array}{r} 2 \text{ ft. } 9 \text{ in.} \\ +4 \text{ ft. } 8 \text{ in.} \\ \hline \end{array}$$

First add the inches $\begin{array}{r} 9 \text{ in.} \\ + 8 \text{ in.} \\ \hline 17 \text{ in.} \end{array}$

$$\begin{array}{r} 1 \text{ ft.} \\ 2 \text{ ft. } 9 \text{ in.} \\ +4 \text{ ft. } 8 \text{ in.} \\ \hline 5 \text{ in.} \end{array}$$

17 in. is more than 1 foot
(1 ft = 12 in.), so write down the 5 inches remaining and carry the 1 foot.

$$\begin{array}{r} 1 \text{ ft.} \\ 2 \text{ ft. } 9 \text{ in.} \\ +4 \text{ ft. } 8 \text{ in.} \\ \hline 7 \text{ ft. } 5 \text{ in.} \end{array}$$

Now, add the feet.

Therefore the answer is 7 ft. 5 in.

ACTIVITY 1 - OUTCOMES 15 and 16 (Cont'd)

Ask students to consider elementary subtraction problems.

$$\begin{array}{r} 84 \\ -57 \\ \hline \end{array}$$

Subtract the units first.

$$\begin{array}{r} 714 \\ \cancel{8}4 \\ -57 \\ \hline \end{array}$$

Borrow 1 ten and add the ten to the 4 in the units place.

$$\begin{array}{r} 714 \\ \cancel{8}4 \\ -57 \\ \hline 27 \end{array}$$

Now subtract.

A similar procedure is used to subtract units of measure.

Example 1: $\begin{array}{r} 8 \text{ ft. } 4 \text{ in.} \\ -5 \text{ ft. } 7 \text{ in.} \\ \hline \end{array}$ Subtract the inches first.

$\begin{array}{r} 7 \text{ ft. } 16 \text{ in.} \\ \cancel{8} \text{ ft. } \cancel{4} \text{ in.} \\ -5 \text{ ft. } 7 \text{ in.} \\ \hline \end{array}$ Borrow 1 foot.
Since 1 foot equals 12 inches we add 12 in. to 4 in. and get 16 in.

$\begin{array}{r} 7 \text{ ft. } 16 \text{ in.} \\ \cancel{8} \text{ ft. } \cancel{4} \text{ in.} \\ -5 \text{ ft. } 7 \text{ in.} \\ \hline 2 \text{ ft. } 9 \text{ in.} \end{array}$ Now subtract.

Consider the algorithm for multiplication next.

$$\begin{array}{r} 28 \\ \times 7 \\ \hline \end{array}$$

Multiply seven times the 8 in the ones place and get 56.

ACTIVITY 1 - OUTCOMES 15 and 16 (Cont'd)

$$\begin{array}{r} 5 \\ 28 \\ \times 7 \\ \hline 6 \end{array}$$

Put down the 6 and carry the 5.

$$\begin{array}{r} 28 \\ \times 7 \\ \hline 196 \end{array}$$

Multiply 7 x 2 in the tens place and get 14. Then add the 5 that was carried to get 19.

Now a similar procedure for multiplying units of measure.

Example 2: 2 lb. 8 oz. Multiply 7 times 8 oz. and get 56 oz.
 $\times \quad 7$ Since 56 oz. is 3 lb. and 8 oz.,

3 lb. 8 oz. Put down the 8 oz. and carry the 3 lb.
 $+14 \text{ lb.}$ Multiply 7 times 2 lb. and get 14 lb.
17 lb. 8 oz.

Therefore, 2 lb. 8 oz. x 7 = 17 lb. 8 oz.

Consider the algorithm for division next.

$$3 \overline{)726}$$

$$\begin{array}{r} 2 \\ 3 \overline{)726} \\ \underline{6} \end{array}$$

Divide 3 into 7, and multiply the trial quotient times divisor.

$$\begin{array}{r} 24 \\ 3 \overline{)726} \\ \underline{6} \\ 12 \\ \underline{12} \end{array}$$

Divide 3 into partial dividend 12, and multiply partial quotient times divisor 3.

ACTIVITY 1 - OUTCOMES 15 and 16 (Cont'd)

$$\begin{array}{r} 242 \\ 3 \overline{)726} \\ \underline{6} \\ 12 \\ \underline{12} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

Subtract and bring down next number. Divide partial quotient by divisor and multiply partial dividend times divisor, then subtract.

Use a similar procedure for dividing units of measure.

$$3 \overline{)7 \text{ yd. } 2 \text{ ft. } 6 \text{ in.}}$$

$$\begin{array}{r} 2 \\ 3 \overline{)7 \text{ yd. } 2 \text{ ft. } 6 \text{ in.}} \\ \underline{6} \\ 1 \text{ yd.} \end{array}$$

Divide divisor (3) into partial dividend (7). Multiply partial quotient (2) times divisor (3) and get (6). Subtract and get remainder 1 yd.

$$\begin{array}{r} 2 \text{ yd.} \\ 3 \overline{)7 \text{ yd. } 2 \text{ ft. } 6 \text{ in.}} \\ \underline{6 \text{ yd.}} \\ 1 \text{ yd.} = 3 \text{ ft.} \\ \underline{ \text{ yd.}} \\ 5 \text{ ft.} \end{array}$$

Convert 1 yd. to 3 feet and add to partial dividend of 2 feet.

$$\begin{array}{r} 2 \text{ yd. } 1 \text{ ft.} \\ 3 \overline{)7 \text{ yd. } 2 \text{ ft. } 6 \text{ in.}} \\ \underline{6 \text{ yd.}} \\ 1 \text{ yd.} = 3 \text{ ft.} \\ \underline{ \text{ yd.}} \\ 5 \text{ ft.} \\ \underline{-3 \text{ ft.}} \\ 2 \text{ ft.} \end{array}$$

Divide divisor of 3 into partial dividend of 5 feet. Get 1 ft. Multiply trial quotient times divisor of 3 and get 3 ft. Subtract and get 2 ft.

ACTIVITY 1 - OUTCOMES 15 and 16 (Cont'd)

$$\begin{array}{r}
 2 \text{ yd.} \quad 1 \text{ ft.} \quad 10 \text{ in.} \\
 3/7 \text{ yd.} \quad 2 \text{ ft.} \quad 6 \text{ in.} \\
 \hline
 6 \text{ yd.} \\
 1 \text{ yd.} = 3 \text{ ft.} \\
 \quad 5 \text{ ft.} \\
 \quad -3 \text{ ft.} \\
 \quad \hline
 \quad 2 \text{ ft.} = 24 \text{ in.} \\
 \quad \quad 30 \text{ in.} \\
 \quad \quad 30 \text{ in.} \\
 \quad \quad \hline
 \quad \quad 0
 \end{array}$$

Since the remainder 2 ft. = 24 in. add the 24 in. to the 6 in. and get 30 in.
 Divide the divisor of 3 into the partial quotient of 30 and get 10.
 Multiply partial quotient of 10 times divisor of 3 and get 30 and subtract.

Give students practice with all types of examples for adding, subtracting, multiplying, and dividing customary and metric units.

ACTIVITY 2 - OUTCOMES 15 and 16

Review adding, subtracting, multiplying, and dividing of units of measure, if necessary. Review converting units of measure, if necessary. Give example problems such as:

1. What unit of measure would be used with apples?
2. What unit of measure would be used to measure a bowl of water?
3. What unit of measure would be used to measure your temperature?
4. Would it be more sensible to measure ice cream in ounces or meters?
5. Would it be more sensible to measure candy in liters or grams?

Give students other word problems such as the following:

ACTIVITY 2 - OUTCOMES 15 and 16 (Cont'd)

Example 1: Mr. Brown needs eight servings of steak. He has a 9 lb. 8 oz. piece of beef. For each person to have an equal share, what size pieces should he cut?

$$\begin{array}{r} 1 \text{ lb. } 3 \text{ oz.} \\ 8/9 \text{ lb. } 8 \text{ oz.} \\ \hline 8 \\ 1 = 16 \\ \hline 24 \\ \hline 24 \end{array}$$

Therefore, Mr. Brown should cut the steaks 1 lb. 3 oz. each.

Example 2: A coach wanted to spend $\frac{2}{3}$ of practice time on one drill. How much time can be spent on one drill if practice lasts for 1 hr. 45 min.?

Method A: Since 1 hr. 45 min. = $1 \frac{45}{60} = 1 \frac{3}{4}$

$$\frac{2}{3} \times 1 \frac{3}{4} =$$

$$\frac{2}{3} \times \frac{7}{4} = \frac{14}{12} = 1 \frac{2}{12} = 1 \frac{1}{6}$$

The answer is $1 \frac{1}{6}$ hr., or 1 hour 10 minutes, since 10 minutes is $\frac{1}{6}$ of 60 minutes.

Method B: Since 1 hr. 45 min. = 105 min.

$$\frac{2}{3} \times \frac{105}{1} = \frac{210}{3} = 70 \text{ min.}$$

Therefore, one drill will be 70 minutes or 1 hr. 10 min.

Example 3: One play dough recipe calls for 1 cup flour, $1 \frac{1}{2}$ cups water, $\frac{1}{2}$ teaspoon cream of tartar, and 2 cups salt. If the recipe is tripled, how much of each ingredient will be needed?

Solution: $1 \text{ c flour} \times 3 = 3 \text{ c flour}$
 $1 \frac{1}{2} \text{ c water} \times 3 = 4 \frac{1}{2} \text{ c water}$
 $\frac{1}{2} \text{ t cream of tartar} \times 3 = 1 \frac{1}{2} \text{ t cream of tartar}$
 $2 \text{ c salt} \times 3 = 6 \text{ c salt}$

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

OUTCOMES: 17. Read thermometers to determine temperature in degrees Fahrenheit and degrees Celsius and solve problems involving temperature.

NOTES FOR THE TEACHER

Prerequisites for Students

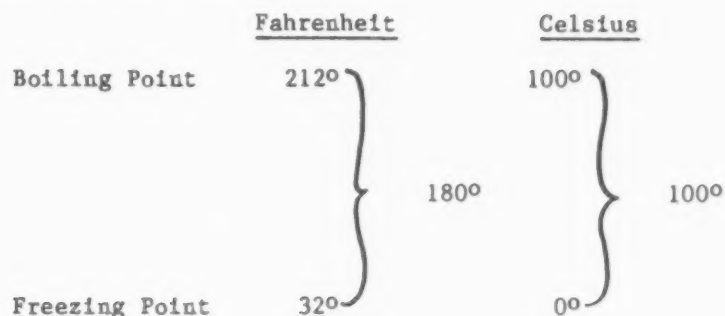
1. Know the basic fractional concepts.
2. Know the symbol used to mean degrees of temperature ($^{\circ}$).

Steps to Success

1. Fahrenheit and Celsius scales have fixed points for boiling and freezing. The scale between the points is different.

ACTIVITY 1 - OUTCOME 17

Draw the following diagram.



Lead a discussion to develop the concept of the difference in the two scales using the numbers 100 and 180.

The Celsius is $180/100$ or $9/5$ of the Fahrenheit scale. The Fahrenheit is $100/180$ or $5/9$ of the Celsius scale.

Let students construct one large thermometer with a Fahrenheit scale on one side and a Celsius scale on the other side. Display it in the room where the temperature of the day can be compared on both scales over a period of several weeks.

ACTIVITY 2 - OUTCOME 17

Have students work in pairs and record the daily high and low temperatures for a week. One student should record in degrees Celsius and the other in degrees Fahrenheit. At the end of the week have the students draw a line graph representing the temperatures they recorded. The graphs for degrees Celsius and degrees Fahrenheit should look alike, even though the actual degrees recorded are different. This activity demonstrates that temperature may be named in at least two different ways.

ACTIVITY 3 - OUTCOME 17

Have students solve problems in which they determine the change in degrees from one time to another and determine whether this change represents a rise or fall in temperature.

Example: A weather observer records a temperature of 75°F at 2:00 p.m. and a temperature of 46°F at midnight.
What is the change in temperature?
Did the temperature rise or fall?

Solution:
$$\begin{array}{r} 75^{\circ} \\ - 46^{\circ} \\ \hline 29^{\circ} \text{ change} \end{array}$$

The 75° temperature was recorded at 2:00 p.m. and the temperature of 46°F was recorded at midnight; therefore, since 46°F is less than 75°F the change represents a fall in temperature.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 12. Measure weight/mass of objects in customary and metric units.
18. Estimate lengths and capacities.
19. Study the history of measurement.
20. Relate history of measurement and applications.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Know the customary units.

Step to Success

1. Emphasize that the metric system is based on tens and is easier to learn than the customary system because it is systematic.

ACTIVITY 1 - OUTCOMES 12, 18, 19, and 20

Explain the following to the students.

Many years ago man had the need for a standard of weight. He took one grain from the very center of the wheat stem. He called this a grain. It took exactly 5760 of these grains to make what we call a pound. Through the years man has used many methods of measuring. At this point, ask the class for examples of different methods of measure such as cloth material measured from the tip of the nose to the finger tips for the yard.

Explain further that as years passed, man changed standards of measure to suit his own business or trade. But for everyone's best interest, a system had to be set so that everyone would get equal measures. One of the simplest systems man developed is called the metric system.

The metric system has three basic units of measure.

1. liter - to measure volume
2. gram - to measure weight
3. meter - to measure length

Compare this to the customary units we use.

- | | |
|------------------------------|--------------------------------|
| 1. liter - to measure volume | - pint
quart
gallon |
| 2. gram - to measure weight | - ounce
pound
ton |
| 3. meter - to measure length | - inch
foot
yard
mile |

Use containers and other measuring devices to illustrate the use of these units of measure.

ACTIVITY 1 - OUTCOMES 12, 18, 19, and 20 (Cont'd)

Make a set of flash cards that ask what unit of measure would be more reasonable to use.

Example 1:

(METRIC UNITS)
A Match Box of Sand

grams

back of card

Example 2:

(CUSTOMARY UNITS)
Length of the Classroom

yards and/or feet

back of card

Example 2:

(METRIC UNITS)
A Humans's Weight

kilograms

back of card

(CUSTOMARY UNITS)
A Cup of Water

ounces

back of card

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 13. Demonstrate an understanding of place value within the metric system.
14. Make conversions among units of length, capacity, or weight/mass within the customary or metric systems.
16. Compute with customary units.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Have a basic knowledge of customary and metric measures.
2. Be able to multiply and divide.
3. Know the abbreviations for both customary and metric measures.

Steps to Success

1. Students must be familiar with the relationship of one customary measure to another. Conversions between the following measures are used frequently:

inches to feet to yards	yards to feet to inches
quarts to gallons	gallons to quarts
ounces to pounds	pounds to ounces

2. Students must know:

12 inches = 1 foot
3 feet = 1 yard
4 quarts = 1 gallon
16 ounces = 1 pound

NOTES FOR THE TEACHER (Cont'd)

3. Students will need to know basic metric information also, mostly with regard to prefixes. Be sure that students notice how all metric conversions use 6 metric prefixes (kilo, hecto, deka, deci, centi, milli) and a base unit (meter, gram, liter).

ACTIVITY 1 - OUTCOMES 14 and 16

CUSTOMARY MEASURES

There are two types of conversion problems. To change from one unit of measure to another, decide if the conversion is (Type 1) from a smaller unit to a larger unit (inches to feet, quarts to gallons, ounces to pounds) or if the conversion is (Type 2) from a larger unit to a smaller unit (feet to inches, gallons to quarts, pounds to ounces).

TYPE 1: FROM A SMALLER UNIT TO A LARGER UNIT.

- (1) Read the problem to determine the unit of measure given. (smaller unit)
- (2) Look for the unit of measure asked for. (larger unit)
- (3) Decide how many of the first or smaller measure it takes to make one of the second measure.

For example: How many inches are in one foot, or how many quarts are in one gallon?

- (4) Divide this number into the number of smaller units given. Division is needed because it takes fewer of a larger unit to equal a given number of smaller units.

Example: 90 feet are equal to _____ yards.

- Solution:
- (1) Determine the unit of measure given. (90 feet)
 - (2) Look for measure you are changing to. (yards)
 - (3) Decide how many feet are in one yard. (3)

ACTIVITY 1 - OUTCOMES 14 and 16 (Cont'd)

CUSTOMARY MEASURES

- (4) Divide 3 into 90 because $\frac{1}{3}$ fewer yards will be needed to equal a given number of feet.
($90 \div 3 = 30$)

90 feet are equal to 30 yards.

Other examples converting from smaller to larger units would be:

48 ounces are equal to _____ pounds.

1 quart is equal to _____ gallons.

In converting from larger units to smaller units, the following strategy is helpful.

TYPE 2: FROM A LARGER UNIT TO A SMALLER UNIT.

- (1) Determine the unit of measure given. (larger unit)
- (2) Look for the unit of measure asked for. (smaller unit)
- (3) Decide how many of the smaller units it takes to make one of the larger unit which was given.

For example: How many inches in one foot or how many quarts in one gallon?

- (4) Multiply this number times the number of the larger units given. Multiplication is needed because it takes more of a smaller unit to equal a given number of larger units.

Example: 30 yards are equal to _____ feet.

Solution: (1) Determine the unit of measure given. (30 yards)

- (2) Look for measure asked for. (feet)
- (3) Decide how many feet are in one yard. (3)
- (4) Multiply 3×30 because three times more feet will be needed to equal a given number of yards.
($3 \times 30 = 90$)
- (5) 30 yards are equal to 90 feet.

ACTIVITY 1 - OUTCOMES 14 and 16 (Cont'd)

CUSTOMARY MEASURES

Other examples converting from larger units to smaller units would be:

2 gallons are equal to _____ quarts.

3 pounds are equal to _____ ounces.

ACTIVITY 2 - OUTCOME 13 and 14

METRIC MEASURES

Point out to students that the metric system is a decimal system of weights and measures. Indicate that the "base" or basic unit will be either a gram (g), meter (m) or liter (L). All other measures are related to the base through multiplication or division by 10. The names of all other measures are related to the base through prefixes. Use the gram (g) as an illustration of how the decimal system measures are named and of the relation of each to the base.

<u>Base</u>	<u>Prefix</u>	<u>Name of Measure</u>	<u>Relation to Base (Number of grams)</u>
gram	kilo -	kilogram	1000
	hecto -	hectogram	100
	deca -	decagram	10
		gram	1
	deci -	decigram	0.1
	centi -	centigram	0.01
	milli -	milligram	0.001

Have students "generate" the names for examples involving liters and meters. For example:

10 x a meter = decameter

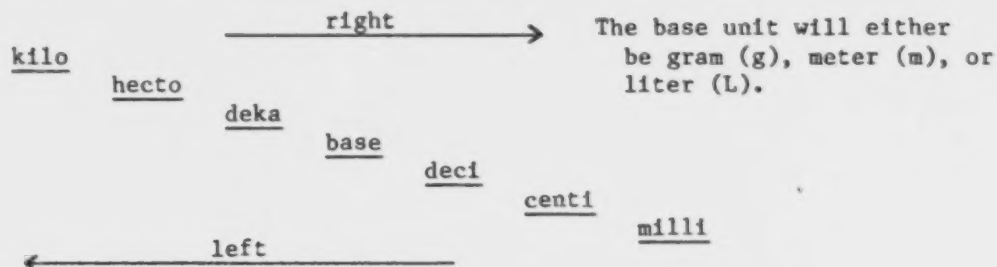
1/100 x a liter =

1000 liters =

ACTIVITY 2 - OUTCOME 13 and 14 (Cont'd)

METRIC MEASURES

Students will benefit from the "stepladder" approach to conversion problems within the metric system. Students will need to memorize the order of prefixes as they are written on the ladder below.



In conversion problems, the decimal point moves according to the steps on the ladder. The direction that the decimal is moved depends upon whether movement on the ladder is up (decimal moves to the left) or down (decimal moves to the right). The number of places that the decimal moves is determined by counting the steps between the first measure and the measures to which one is changing.

Example: 2.3 kilograms is equal to _____ grams.

Solution: (1) The gram, as the base, is three steps down the ladder from kilograms.

(2) Going down the ladder to the right three steps means the decimal moves three places to the right.

(3) Zeros must be added to the right of the decimal for the decimal to be moved three places.

2.3 — add zeros — 2.300

(4) Now, move the decimal three places. (2300)

2.3 kilograms is equal to 2300 grams.

ACTIVITY 2 - OUTCOME 13 and 14 (Cont'd)

METRIC MEASURES

Provide examples like the following for student practice.

900 g = ___ kg (move decimal to left 3 places - .9)
0.8 kg = ___ g (move decimal to right 3 places - 800)
98 kl = ___ l (move decimal to right 3 places - 98000)
475 g = ___ kg (move decimal to left 3 places - .475)
3 ml = ___ dl (move decimal to left 2 places - .03)
3 dm = ___ ml (move decimal to right 2 places - 300)
4 hm = ___ mm (move decimal to right 5 places - 400,000)

Students can make up examples like the ones above to exchange with their classmates.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

OUTCOMES: 21. Find the perimeters of polygons.
22. Solve problems involving perimeters.

NOTES FOR THE TEACHER

Prerequisite for Students

The activity for these outcomes is found also in GENERAL MATHEMATICS - MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS - OUTCOMES 10-15.

1. Have a working knowledge of the four basic operation of whole numbers, decimals, and fractions.

Steps to Success

The key to success is being able to identify the correct formula and doing exactly what the formula indicates. The approach to finding perimeter, however, is to first see perimeter as the sum of the lengths of the sides. Then, an alternative formula is introduced as another way of finding the perimeter. For example, perimeter of a regular hexagon is first seen as $s+s+s+s+s+s$, and then as $6s$.

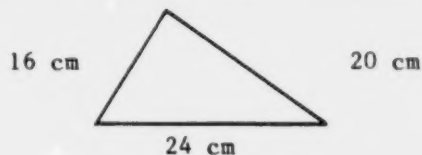
ACTIVITY 1 - OUTCOMES 21 and 22

Formulas are like directions that tell how to put something together. The first thing to do is read the directions. Next, look at the pictures of all the parts. When reading a mathematical formula, look at all the variables, numerals, and signs of operation. What do the variables stand for? What operation is needed to complete the problem? When formulas (directions) are followed, and when the appropriate values for each variable are substituted, then the problem can be solved.

ACTIVITY 1 - OUTCOMES 21 and 22 (Cont'd)

Continue with a discussion of perimeter. On the bulletin board, form a triangle using three tacks and yarn. The distance around it can be found by measuring the amount of string needed to go around the three tacks. Let students form other plane figures and show the distance around them. Ask students if there are other ways to find the distance around each figure. Explain that the distance around figure is called perimeter and that the perimeter can be found by adding the length of all the sides.

Example 1: Draw a triangle with 3 unequal sides.



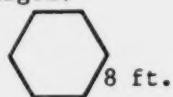
Solution 1: Point out that the perimeter can be found by adding the 3 sides.

$$p = s + s + s$$

$$p = 16 \text{ cm} + 20 \text{ cm} + 24 \text{ cm}$$

$$p = 60 \text{ cm}$$

Example 2: Draw a regular hexagon.



Solution 2: $p = s + s + s + s + s + s$

$$p = 8 \text{ ft.} + 8 \text{ ft.} + 8 \text{ ft.} + 8 \text{ ft.} + 8 \text{ ft.} + 8 \text{ ft.}$$

$$p = 48 \text{ ft.}$$

As a "short cut" indicate that there are six equal sides of 8 ft. each. Or,

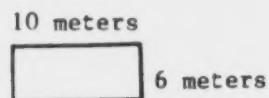
$$p = 6 \times s$$

$$p = 6 \times 8$$

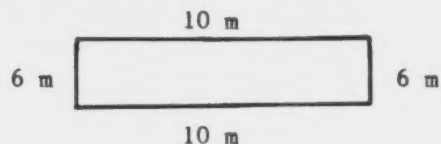
$$p = 48 \text{ ft.}$$

ACTIVITY 1 - OUTCOMES 21 and 22 (Cont'd)

Example 3: How much fencing is needed to enclose a rectangular garden that measures 10 meters by 6 meters?
Draw a picture of the fence.



Solution 3: Point out that students need to label all 4 sides, if they are to find the perimeter.



$$\begin{aligned} p &= s + s + s + s \\ p &= 10 \text{ m} + 6 \text{ m} + 10 \text{ m} + 6 \text{ m} \\ p &= 32 \text{ m} \end{aligned}$$

As an alternative, students may see that half of the perimeter of the rectangle is $10 \text{ m} + 6 \text{ m}$. Therefore, if that half of the perimeter is doubled, the total is generated.

$$\begin{aligned} p &= 2(l + w) \\ p &= 2(10 + 6) \\ p &= 32 \text{ m} \end{aligned}$$

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT AND MONEY

OUTCOME: 23. Find circumference of circles.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Have a working knowledge of the four basic operations of whole numbers, decimals, and fractions.

Steps to Success

1. The key to success is being able to identify the correct formula for finding the circumference of a circle and doing exactly what the formula indicates.
2. The students should know the value of π as a decimal and as a fraction.

ACTIVITY 1 - OUTCOMES 23

Formulas are like directions that tell how to put something together. The first thing to do is read the directions. Next, look at the pictures of all the parts. When reading a mathematical formula, look at all the variables, numerals, and signs of operation. What do the variables stand for? What operation is needed to complete the problem? When formulas (directions) are followed and when the appropriate values for each variable are substituted, then the problem can be solved.

ACTIVITY 1 - OUTCOME 23

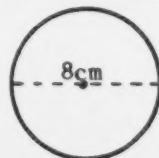
The distance around a circle is not called perimeter. It is called circumference. On the bulletin board place 3 lids. Cut a piece of string so that it just encircles one lid. Point out what happens when that string is stretched across the center of 3 adjacent lids. The illustration at the board shows that the circumference of one lid is slightly greater than the diameters of 3 lids.



Ask students if they think the same thing would happen if lids of different diameters were used. (Yes) Ask students what would happen if the distance around a circle were measured and then that distance were divided by its diameter? Lead students to see that the result would be a little more than 3. Inform students that the Greek letter π (pi) is used to show the relationship between the circumference and the diameter. The decimal 3.14 or the fraction $\frac{22}{7}$ are used as approximations for π . Thus the formula $C = \pi d$.

circumference = $\pi(3.14) \times$ diameter

Example 1: Find the circumference of the circle.



Solution 1: $C = \pi d$
 $C = 3.14 \times 8\text{cm}$
 $C = 25.12\text{cm}$

ACTIVITY 1 - OUTCOME 23 (Cont'd)

Example 2: Find the circumference of the circle.



Solution 2: $C = \pi d$

$$C = 22/7 \times 5 \frac{1}{4}$$

$$C = \frac{22}{7} \times \frac{21}{4}$$

$$C = 33/2$$

$$C = 16 \frac{1}{2} \text{ft.}$$

NOTE: Emphasize to students that had they been give a radius rather than a diameter each example could be worked using the formula $C = 2\pi r$ and the results would be exactly the same. The $2r$ in the formula is equivalent to the diameter.

Example 3: Find the circumference of the circle.



Solution 3: $C = 2\pi r$

$$C = 2 \times 3.14 \times 4$$

$$C = 25.12 \text{ cm}$$

The same as Example 1.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

OUTCOMES: 24. Solve problems involving area of geometric figures.
25. Find surface area of prisms and cylinders.

NOTES FOR THE TEACHER

The activities for these outcomes are found also in GENERAL MATHEMATICS - MEASUREMENT INVOLVING BOTH THE METRIC AND CUSTOMARY UNITS - OUTCOMES 5 and 6.

Prerequisites for Students

1. Perform the four basic operations with whole numbers, fractions, and decimals.
2. Identify plane figures as well as prisms.

Steps to Success

1. The ability to interpret and use formulas will be helpful.
2. Emphasize that area is always some version of length times width and is always expressed in square units.

ACTIVITY 1 - OUTCOMES 24 and 25

Have students read the following note:

Go home this afternoon after school.
Eat the cake I have left for you.

ACTIVITY 1 - OUTCOMES 24 and 25 (Cont'd)

Mow the neighbor's yard.
Collect \$15.00 if you finish.

Ask students to analyze the message.

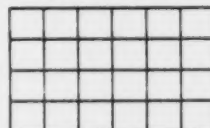
It tells someone what to do step-by-step. What are the consequences for leaving out a step?

A formula is a message. It tells exactly what to do with the values of variables in order to achieve a task.

Consider the following messages as the 6 area formulas are discussed.

Message: To find the AREA OF A RECTANGLE, multiply length times width.

1. RECTANGLE



4 in.

6 in.

$$A = l \times w$$

$$A = 6 \times 4$$

$$A = 24 \text{ sq. in.}$$

Message: To find the AREA OF A SQUARE, multiply side times side.

2. SQUARE



3 cm

3 cm.

$$A = s^2$$

$$A = 3^2$$

$$A = 3 \times 3$$

$$A = 9 \text{ sq. cm}$$

Message: To find the AREA OF A PARALLELOGRAM, multiply base times height.

3. PARALLELOGRAM



3 ft.

6 ft.

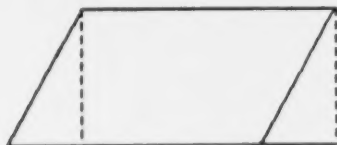
$$A = b \times h$$

$$A = 6 \times 3$$

$$A = 18 \text{ sq. ft.}$$

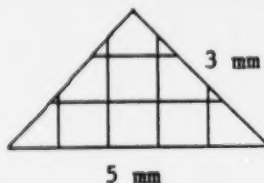
ACTIVITY 1 - OUTCOMES 24 and 25 (Cont'd)

Show how a parallelogram has the same area as a rectangle.



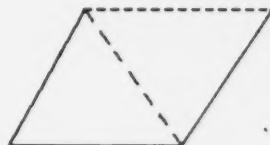
Message: To find the AREA OF A TRIANGLE take $1/2$ of the product of base and height.

4. TRIANGLE



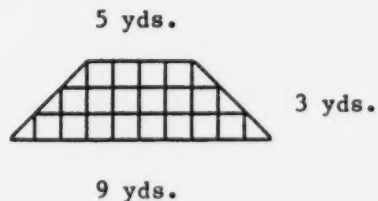
$$\begin{aligned} A &= 1/2 \times b \times h \\ A &= 1/2 \times 5 \times 3 \\ A &= 15/2 \\ A &= 7 \frac{1}{2} \text{ sq. mm} \end{aligned}$$

Show a triangle is $1/2$ a parallelogram.



Message: To find the AREA OF A TRAPEZOID take $1/2$ the sum of the bases and multiply by the height.

5. TRAPEZOID



$$\begin{aligned} A &= \frac{B + b}{2} \times h \\ A &= 1/2 \times (B + b) \times h \\ A &= 1/2 \times (9 + 5) \times 3 \\ A &= 1/2 \times (14) \times 3 \\ A &= 7 \times 3 \\ A &= 21 \text{ sq. yds.} \end{aligned}$$

ACTIVITY 1 - OUTCOMES 24 and 25 (Cont'd)

Show how a trapezoid is $1/2$ a parallelogram.



Message: To find the AREA OF A CIRCLE, multiply π times the square of the radius.

6. CIRCLE



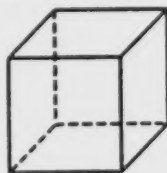
$$\begin{aligned} A &= \pi \times R^2 \\ A &= 3.14 \times 5^2 \\ A &= 3.14 \times 25 \\ A &= 78.5 \text{ sq. in.} \end{aligned}$$

ACTIVITY 2 - OUTCOME 24 and 25

Explain that the surface area is the part of a prism, pyramid, cylinder, cone, or sphere that would be covered if the surfaces of a solid figure were to be printed.

Let the students help develop the message and the formula for the surface area of a cube, a rectangular solid, and a pyramid.

1. Message: The surface area of a cube is the area of one side multiplied by 6. (6 sides)



$$A = 6s^2$$

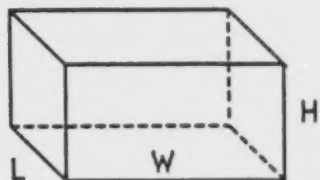
Example: Find the surface area of a cube whose side is 3 decimeters.

$$\begin{aligned} A &= 6(s^2) \\ A &= 6 \times 3^2 \\ A &= 6 \times 9 \\ A &= 54 \text{ sq. dm} \end{aligned}$$

ACTIVITY 2 - OUTCOMES 24 and 25 (Cont'd)

2. Message: The surface area of a rectangular solid is the area of the top and bottom plus the area of the two sides plus the area of the two ends.

Rectangular Solid



$$A = 2lw + 2lh + 2wh$$

Bottom and top	Two sides	Both ends
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Example: Find the surface area of a rectangular solid whose length is 6 ft., width is 4 ft., and height is 2 ft.

Solution:

$$A = 2lw + 2lh + 2wh$$

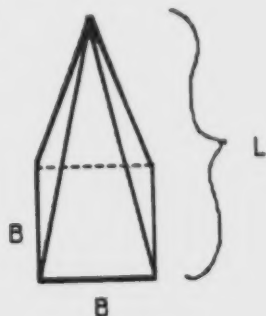
$$A = 2(6)(4) + 2(6)(2) + 2(4)(2)$$

$$A = 48 + 24 + 16$$

$$A = 88 \text{ sq. ft.}$$

3. Message: The surface area of a pyramid is the area of its base plus the area of its four sides.

Pyramid



$$A = b^2 + 4(1/2 bh)$$

Example: Find the surface area of a pyramid whose base is 3 m and height is 4 m.

Solution:

$$A = b^2 + 4(1/2 bh)$$

$$A = 3^2 + 4(1/2 \times 3 \times 4)$$

$$A = 9 + 4(6)$$

$$A = 33 \text{ sq. m}$$

ACTIVITY 2 - OUTCOMES 24 and 25 (Cont'd)

4. Message: The surface area of a cylinder is the area of the 2 circles that serve as its top and bottom ($2\pi r^2$) plus the area of the circular wall that would be a rectangle if it were cut and laid flat. The rectangle has a length of πd or $2\pi r$ (circumference of a circle) and a height of h .



Example: Find the surface area of a cylinder whose height is 6 yds. and radius of the base is 2 yds.

Solution: The formula for the surface area of a cylinder is:

$$A = 2\pi r^2 + 2\pi r(h)$$

Rewritten by factoring out the $2\pi r$, the formula for the surface area of the cylinder becomes:

$$A = 2\pi r(h + r)$$

Apply the formula.

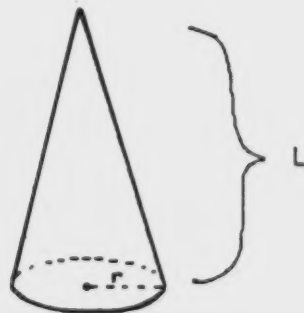
$$A = 2\pi r(h + r)$$

$$A = 2(3.14)(2)(6 + 2)$$

$$A = 2 \times 3.14 \times 2 \times 8$$

$$A = 100.48 \text{ sq. yds.}$$

5. Message: The surface area of a cone is found by using the formula $A = \pi r^2 + \pi r l$



Example: Find the surface area of a cone whose slant height (l) is 5 mm and the radius of the base is 1.5 mm.

Solution:

$$A = \pi r^2 + \pi r l$$

$$A = (3.14)(1.5)^2 + 3.14(1.5)(5)$$

$$A = 3.14 \times 2.25 + 3.14 \times 7.5$$

$$A = 7.065 + 23.55$$

$$A = 30.615 \text{ sq. mm}$$

ACTIVITY 2 - OUTCOMES 24 and 25 (Cont'd)

6. Message: The surface area of a sphere is found by using the formula $A = 4\pi r^2$



Example: Find the surface area of a sphere whose radius is 12 meters.

Solution: $A = 4\pi r^2$
 $A = 4 \times 3.14 \times 12^2$
 $A = 4 \times 3.14 \times 144$
 $A = 1808.64 \text{ sq. m}$

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 26. Find the volume of prisms and cylinders.
27. Find volumes to include prisms, cylinders, pyramids, cones, and spheres.

AREA: APPLICATIONS

- OUTCOME: 4. Solve word problems about realistic situations requiring the use of measurement skills.

NOTES FOR THE TEACHER

Prerequisites for Student

1. Perform the four basic operations with whole numbers, fractions, and decimals.
2. Identify as prisms, cylinders, pyramids, cones, and spheres.

Steps to Success

1. The ability to interpret and use formulas will be helpful.
3. Emphasize that volume is some version of length times width times height. Volume is always found in cubic units.

ACTIVITY 1 - TIME, MEASUREMENT, AND MONEY - OUTCOMES 26 and 27
APPLICATIONS - OUTCOME 4

Have students read the following note:

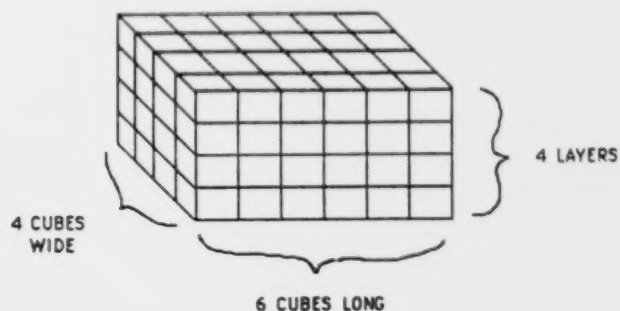
Go home this afternoon after school.
Eat the cake I have left for you.
Mow the neighbor's yard.
Collect \$15.00 if you finish.

Ask students to analyze the message.

It tells someone what to do step-by-step. What are the consequences for leaving out a step?

A formula is a message. It tells exactly what is to be done with the values of variables in order to achieve a task.

The formula for finding volume is derived by counting the number of cubes in a solid figure. An example is a rectangular solid that is $4 \times 6 \times 4$.



$4 \times 6 = 24$ cubes in one layer.

$24 \times 4 = 96$

96 cubes in 4 layers.

Therefore, $v = l \times w \times h$

NOTE: All volume is measured in cubic units.

ACTIVITY 1 - TIME, MEASUREMENT, AND MONEY - OUTCOMES 26 and 27 (Cont'd)
APPLICATIONS - OUTCOME 4

The following are formulas for finding the volume of these geometric solids.

Cube $v = s^3$
or $v = s \times s \times s$

Cylinder $v = \pi r^2 h$

Sphere $v = \frac{4}{3} \pi r^3$

Cone $v = \frac{1}{3} \pi r^2 h$

Pyramid $v = \frac{1}{3} Bh$, where B = area of the base

Give examples such as:

1. What is the volume of an engine cylinder whose radius is 2 in. and whose height is 5 in.?

$$\begin{aligned}v &= \pi r^2 h \\v &= 3.14 \times 2^2 \times 5 \text{ in.} \\v &= 3.14 \times 4 \times 5 \text{ in.} \\v &= 62.80 \text{ cu. in.}\end{aligned}$$

2. How many cubic feet of space are inside a square, pyramid-shaped tent whose base is 10 ft. and whose height is 15 ft.?

$B = s^2$	$V = \frac{1}{3} Bh$
$B = 10^2$	$V = \frac{1}{3}(100\text{ft.})15\text{ft.}$
$B = 100$	$V = 500 \text{ cu. ft.}$

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 8. Solve problems involving rate, time, distance, and miles per gallon.
28. Continue work with determining, making, and counting change from practical application problems.
29. Solve problems that require determination of change after a purchase.
30. Complete information on a check and use this information in learning about checking accounts.
31. Extend work on checks and money orders to include work on balancing checking and savings accounts.
32. Identify and complete information on a money order.
33. Determine amount of tax on purchases.
34. Read a sales tax table to determine tax on a purchase.
35. Figure simple income tax using tables and charts.
36. Compute the amount of discount on a purchase.
37. Solve problems with unit and multiple pricing to compare prices.
43. Use information from budgets to solve problems.
44. Solve problems involving allocation of time and money.

AREA: APPLICATIONS

- OUTCOME: 2. Solve word problems related to consumers to include discounts, down payments, commission, taxes, ratios, and percents.

NOTES FOR THE TEACHER

These outcomes are discussed thoroughly in the mathematics section of the Alabama High School Graduation Exam Basic Skills Instructional Materials (BSIM). Any further development of these outcomes would be a natural extension of the materials contained in that document.

- 8. BSIM - Competency VI-6
- 28. BSIM - Competency VI-1
- 29. BSIM - Competency VI-1
- 30. BSIM - Competency VI-5
- 31. BSIM - Competency VI-5

NOTES FOR THE TEACHER (Cont'd)

- 32. BSIM - Competency VI-5
- 33. BSIM - Competency I-7
- 34. BSIM - Competency III-2
- 35. BSIM - Competency III-3
- 36. BSIM - Competency
- 37. BSIM - Competency VI-2
- 43. BSIM - Competency III-1
- 44. BSIM - Competency VI-7

APPLICATIONS

- 2. BSIM - Competency I-7

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 38. Find amounts of commission.
41. Compute wages based on hourly earnings and overtime.
42. Determine the amount of earnings based on salary, fees, and hourly wages with or without deductions.

NOTES FOR THE TEACHER

Prerequisite for Students

1. Add, subtract, and multiply whole numbers and decimals.

Steps to Success

Review the following terms:

Overtime - time worked over 40 hours per week

Gross pay or salary - total amount of money earned before deductions

Net pay or take home pay - amount of money left after deductions

Deductions - amounts of money subtracted from gross pay

Type of deductions - FICA, federal tax, state tax, retirement, State Unemployment Insurance (S.U.I.)

Bonus - Extra money paid for exceptionally good work

ACTIVITY 1 - OUTCOMES 41 and 42

Ask students to relate any personal working situations involving wages. In addition, ask students to bring in pay stubs if available from older members of their families. (The teacher may also bring in actual stubs for students to review and discuss.)

From the information above, identify examples of gross pay, net pay, deductions, overtime, hourly wages.

ACTIVITY 2 - OUTCOMES 41 and 42

In some problems, hourly wage is given and total hours worked is also provided. Total earnings can then be found by multiplying these two numbers together.

Example: Mona is paid \$4.50 per hour for typing. What is her weekly salary if she types 35 hours per week?

Solution: (1) The pay per hour is \$4.50 \$4.50
(2) Total hours worked is 35 x 35
 2250
(3) Weekly salary is \$157.50 1350
 \$157.50

ACTIVITY 3 - OUTCOMES 41 and 42

Introduce students to examples in which deductions must be subtracted from wages.

In these problems the computation of wages has already been done. Students must find the amount of deductions and subtract this amount from the gross pay or salary. If total deductions are not given, all deductions are added together and subtracted.

Example 1: Jeff's gross pay is \$280. His employer takes out \$52 for federal tax. Find Jeff's net pay.

ACTIVITY 3 - OUTCOMES 41 and 42 (Cont'd)

- Solution:
- (1) The amount of gross pay or salary is \$280.
 - (2) The amount of deductions is given as a total of \$52.
 - (3) Subtract deductions from gross pay to find net pay.

$$\begin{array}{r} \$280 \\ - 52 \\ \hline \$228 \end{array}$$

Example 2: Janice's weekly salary is \$250. Her deductions are FICA \$20, federal tax \$45, and state tax \$22. What is her take-home pay?

- Solution:
- (1) Find the gross pay or salary. (\$250)
 - (2) Find the total amount of deductions: FICA = \$20, federal tax = \$45, state tax = \$22.

$$\begin{array}{r} \$20 \\ 45 \\ 22 \\ \hline \$87 \end{array} \text{ - Total deductions}$$

- (3) Subtract deductions from gross pay to find her take-home pay. (\$163)

$$\begin{array}{r} \$250 \\ - 87 \\ \hline \$163 \end{array}$$

ACTIVITY 4 - OUTCOMES 41 and 42

In this problem the gross pay is not given. You must use addition and multiplication skills to find it. To find the gross pay, multiply the number of hours worked times the hourly rate of pay. If different rates are given for hours of overtime worked, multiply the number of overtime hours times their rate. Then add this amount to the hourly wages total to find the gross pay.

To find the net or take-home pay, subtract the total deductions from the gross pay.

ACTIVITY 4 - OUTCOMES 41 and 42 (Cont'd)

NOTE: If the deductions are not totaled, find the sum of all deductions and then subtract.

Example: Norma works 40 hours a week at \$4.00 an hour. She also works 12 hours overtime at \$6.00 per hour. What is her take-home pay if her deductions are \$65.00?

Solution: (1) Find the hours worked at regular hourly rate. Multiply hours times rate.

$$\begin{array}{r} 40 \text{ hours} \\ \times 4 \text{ dollars an hour} \\ \hline \$160 \text{ regular earnings} \end{array}$$

(2) Find the overtime hours worked at overtime rate. Multiply hours times rate.

$$\begin{array}{r} 12 \text{ hours} \\ \times 6 \text{ dollars an hour} \\ \hline \$72 \text{ overtime pay} \end{array}$$

(3) Add amount of regular earning and amount of overtime earning together. The sum is the total gross earnings.

$$\begin{array}{r} \$160 \\ + 72 \\ \hline \$232 \text{ total gross earnings} \end{array}$$

(4) Subtract deductions (\$65) from gross earnings. The answer is Norma's take-home pay.

$$\begin{array}{r} \$232 \\ - 65 \\ \hline \$167 \text{ take-home pay} \end{array}$$

ACTIVITY 5 - OUTCOMES 38 and 41

This activity requires students to compute wages based on hourly earnings and overtime and to compare these with earnings based on commission. Provide situations like the one described below for students to work on.

Fred, a carpenter, is paid on the basis of hourly earnings plus overtime for work in excess of 140 hours a month. His wife, Joan, a real estate broker, is paid a monthly commission on all gross sales completed. Fred receives \$18.50 per hour worked and time and a half for hours worked in excess of 140 per month. Joan is paid a straight 5% commission on all gross sales completed each month.

Listed below are the number of hours each month that Fred worked during the past year, and the amount of gross sales each month that his wife, Joan, completed.

<u>Fred's Hours</u>		<u>Joan's Sales</u>
January	56	\$ 15,000.00
February	115	20,000.00
March	120	35,000.00
April	175	100,000.00
May	200	150,000.00
June	220	175,000.00
July	140	85,000.00
August	160	25,000.00
September	120	0
October	80	15,000.00
November	10	0
December	10	0

Have students determine how many regular and overtime hours Fred worked. Tell students to compute by month how much Fred earned and his total yearly earnings.

Have students determine by month how much Joan earned and her total yearly earnings based on 5% commission of gross sales.

Class discussion should center around a comparison of the earnings of Fred and Joan and hourly wages versus straight commission.

ACTIVITY 5 - OUTCOMES 38 and 41 (Cont'd)

As an extension, have students list occupations in which wages are earned on an hourly and overtime basis. Occupations in which wages are based on commission should be listed also. Have students identify a person they know who works in one of the occupations listed. Let each student conduct an interview with the person they have identified. During this interview, information should be obtained which describes typical wages earned by persons working in that occupation.

MATHEMATICS
GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

- OUTCOMES: 39. Use knowledge of installment buying and down payments to compare cash and credit purchases.
40. Compare cash and credit purchases to include computing down payments and interest.

NOTES FOR THE TEACHER

Prerequisites for Students

1. Add, subtract, multiply, and divide whole numbers and decimals.
2. Change percents to decimals.

Steps to Success

1. Review common terms that may appear in these problems: for example, credit, interest, installment, down payment, balance, cash purchase, credit purchase, cash savings, and monthly payments.
2. Emphasize to students that credit and installment buying will require the buyer to pay more money than buying with cash.
3. Include problems comparing cash to credit purchases and problems involving interest charged on unpaid balances.

ACTIVITY 1 - OUTCOMES 39 and 40

Introduce students to situations in which it is possible to pay cash or to pay in monthly installments. Work several examples together and emphasize the circumstances which cause a person to pay either the total amount in one payment (cash purchase) or in monthly installments (credit purchase).

ACTIVITY 1 - OUTCOMES 39 and 40 (Cont'd)

Example:

STEREO \$160 cash or \$20 down and \$12.50 a month for 12 months

Hal is buying the stereo listed in the ad. If he decides to pay cash for the stereo, how much will he save?

Solution: Hal knows he will have to pay \$160 if he pays cash. To determine how much he will pay if he buys on credit:

- | | | |
|--|---|---|
| (1) Multiply the amount he will pay each month times the number of months he will pay. | $\begin{array}{r} \$ 12.50 \\ \times 12 \\ \hline \$150.00 \end{array}$ | <p>amount per month
number of months
monthly payments</p> |
| (2) Add the amount <u>paid down</u> to the above answer. This is the amount he will pay if he buys the stereo on credit. | $\begin{array}{r} \$150.00 \\ +20.00 \\ \hline \$170.00 \end{array}$ | <p>monthly payments
down payment
total credit cost</p> |
| (3) The amount he will save is the difference between the credit cost and the cash cost. | $\begin{array}{r} \$170.00 \\ -160.00 \\ \hline \$ 10.00 \end{array}$ | <p>credit cost
cash cost
cash savings</p> |

ACTIVITY 2 - OUTCOMES 39 and 40

Have students describe verbally or in writing a situation where they have the option of buying an item with one payment or with a down payment and monthly installments. Be sure that students include all necessary information in their descriptions. Arrange for these real life situations to be exchanged and discussed. Have students compare the cash purchase with the credit purchase in each situation.

ACTIVITY 3 - OUTCOMES 39 and 40

Introduce students to situations in which an interest charge is added to a purchase made over a period of time.

Example: Jane's clothing bill for March showed an outstanding balance of \$125.00. She was charged 2% on the outstanding balance. How much interest did she have to pay?

Solution: To find the interest, first change the 2% to a decimal. (2% = .02)

Then multiply the outstanding balance by the interest.

$$\begin{array}{r} \$125.00 \\ \times .02 \\ \hline \$2.5000 \end{array}$$

She had to pay \$2.50 interest.

ACTIVITY 4 - OUTCOME 40

Junior high school students will not normally purchase high cost items on long-term credit. This activity is designed to alert them to interest charges on long-term credit purchases. Advise students that during their life times, they will probably purchase a house, automobile, color TV, video cassette machine and perhaps a boat. To buy these items, they may need to obtain long-term credit.

Have students decide upon an item they believe they will want to purchase. Instruct them that for the purpose of this activity, they will visit a place where the item is sold (for example, automobile dealership, real estate agent, marine sales, etc.). They are to inquire about the cost of purchasing the item by time payments and obtain the following information:

1. Item to be purchased
2. Purchase price
3. Required down payment
4. Amount to be financed
5. Number of months over which repayment is to be made
6. Amount of interest to be paid
7. Monthly payments (including interest)
8. Total cost of purchase (down payment plus monthly payments)
9. Difference between cash purchase and purchase over time.

Students should report to class what they have learned about paying interest on long-term credit purchases.

MATHEMATICS
GRADES 7 and 8

AREA: APPLICATIONS

OUTCOME: 3. Solve word problems that involve geometry skills.

NOTES FOR THE TEACHER

Although activities for GEOMETRY-OUTCOMES 1-13 do not address APPLICATIONS-OUTCOME 3, per se, geometry problems are, indeed, word problems.

In this vein, OUTCOME 3 has been addressed through activities for GEOMETRY-OUTCOMES 1-13.

MATHEMATICS: GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

C O F F	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
7 8	7,8		Demonstrate an understanding of the system of whole numbers and place value by recognizing names for numbers, reading and interpreting numerals, and rounding numbers				
	7,8		Demonstrate an understanding of the fundamental operations and their properties				
	7,8		Solve problems, express problems as number sentences, and identify missing or irrelevant data				
		I-1	Read and write numerals and word names for numerals				
				1. Read and write money values to \$10,000.00 and whole numbers to seven digits.			
				2. Read and write money values and whole numbers to nine digits.			
				3. Demonstrate an understanding of place value.			
				4. Correctly use signs of operations.			
				5. Correctly use order of operations.			
				6. Translate word sentences to number sentences and vice versa.			
				7. Translate and solve number sentences.			

MATHEMATICS: GRADES 7 and 8

AREA: NUMBERS - NUMERATION - NUMBER THEORY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
		7,8		8. Solve open sentences, equalities, and inequalities.			
8				9. Begin study of systems of numbers.			
7				10. Construct and use a number line to demonstrate signed numbers—integers.			
8				11. Use vocabulary and properties of integers to perform operations with whole numbers.			
7				12. Use exponents to indicate the square of a number. Calculate the product.			
8				13. Use exponents and extend to finding the root of a number.			
9				14. Extend exponential use to finding powers and roots of perfect squares and cubes.			
7,8				15. Write numbers in expanded notation and use powers of ten (10) to write numbers in scientific notation.			
9				16. Use scientific notation for expressing extremely large and small numbers. (example: metric measure)			

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MATHEMATICS: GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

C O F F	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	7,8		Demonstrate an understanding of the fundamental operations and their properties.				
	7,8		Multiply by one-, two-, and three-digit factors				
	7,8		Divide by one-, two-, and three-digit divisors				
	7,8		Estimate products and quotients of whole numbers and decimal numbers				
	7,8		Demonstrate an understanding of the fundamental operations and their properties				
	7,8		Solve problems, express problems as number sentences, and identify missing or irrelevant data				
	7,8		Demonstrate the ability to solve proportions and linear equations				
		I-3	Add, subtract, multiply, and divide whole numbers				
		IV-1	Make approximations by rounding				
		I-9	Find averages of whole numbers and decimal numbers				

MATHEMATICS: GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
			Computation				
7,8				1. Add and subtract whole numbers to seven digits with regrouping.			
7,8	7,8			2. Multiply and divide with multipliers and divisors up to three digits.			
7,8				3. Multiply and divide with zero and one in multipliers, divisors, and dividends.			
7,8				4. Give examples of the properties of addition and multiplication and order of operations.			
7,8				5. Use addition and division to find averages of whole and decimal numbers (word problems may be used).			
			Approximations				
7,8				6. Estimate sums, differences, products, and quotients to computational problems.			
				7. Solve problems using approximations to find sums, differences, products, and quotients.			
9		IV-2		8. Select reasonable answers to problems using distance, capacity, and weight.			
			Operations and Properties				
7,8				9. Identify composite and prime numbers and write the prime factorization of numbers.			

MATHEMATICS: GRADES 7 and 8

AREA: OPERATIONS WITH WHOLE NUMBERS

C O F F	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
7,8				10. Determine least common multiple (LCM) and greatest common factor (GCF).			
7				11. Recognize number patterns and relationships.			
8				12. Develop and use number patterns in simple functions and series.			
			Equations				
7,8				13. Solve horizontal equations.			

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MATHEMATICS: GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	7,8		Interpret place value in numerals of up to six digits and round numbers				
	7,8		Name, order, and interpret fractions, decimals, percents, and integers				
	7,8		Add, subtract, multiply, and divide with fractions				
	7,8		Add, subtract, multiply, and divide with decimals				
	7,8		Estimate products and quotients of whole numbers and decimal numbers				
		I-2	Compare decimals; compare fractions with unlike denominators				
		I-4	Add, subtract, multiply, and divide decimals				
		I-5	Add, subtract, and multiply fractions				
		I-6	Change percents to decimals and decimals to percents				
		I-7	Convert fractions to decimals and decimals to fractions				
		I-8	Find averages of whole numbers and decimal numbers				

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MATHEMATICS: GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
7,8			Compare and Order	1. Order and compare decimals and fractions with unlike denominators.			
7				2. Review decimal place value to thousandths.			
8				3. Review decimal place value to ten-thousandths.			
7,8			Computation	4. Add, subtract, multiply, and divide decimals and fractions.			
7				5. Solve problems using decimals larger than one and mixed numbers.			
8				6. Solve problems with metric and money values.			
7				7. Solve word problems involving decimals and fractions.			
8				8. Solve word problems involving decimals, fractions, and ratios and proportions.			
9				9. Solve problems involving addition, subtraction, and multiplication of fractions.			
7,8			Estimation	10. Round decimals.			
7,8				11. Estimate sums, differences, products, and quotients.			

MATHEMATICS: GRADES 7 and 8

AREA: FRACTIONS AND DECIMALS

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
			Conversion	12. Write equivalent fractions. 13. Recognize fractions as ratios, and equivalent fractions as proportions. 14. Use equivalent fractions to solve proportion problems. Use both ratio symbols (e.g. $\frac{2}{3}$ and 2:3) 15. Convert fractions to decimals and vice versa.			

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MATHEMATICS: GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	7,8		Interpret percents				
	7,8		Demonstrate the ability to compute with percents				
	7,8		Demonstrate the ability to solve proportions and linear equations				
		I-6	Change percents to decimals and decimals to percents				
		I-7	Determine discounts, down payments, commissions, taxes, ratios, and percents				
			Ratios and Proportions				
7,8				1. Write and use ratios to compare one quantity to another.			
7				2. Write a proportion using two equal ratios.			
7,8				3. Solve proportions.			
8				4. Estimate solutions to proportions.			
7,8				5. Use proportions to solve word problems.			
8				6. Use proportions to solve word problems involving rates or scale drawings.			

MATHEMATICS: GRADES 7 and 8

AREA: RATIO, PROPORTION, AND PERCENT

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
			Percent				
7,8				7. Demonstrate an understanding of percent terminology e.g. percent, percentage, and the percent symbol (%).			
7,8				8. Change percents to decimals and decimals to percents.			
9				9. Recognize percent as a ratio.			
7,8				10. Solve number problems for: a percent of a number, what percent one number is of another number, and the number if a percentage of the number is known.			
7,8				11. Calculate discounts, simple interest, and interest rates, mark-ups, tax, and other business uses of percent.			
9				12. Recognize Trigonometric ratios - sine, cosine, and tangent (optional).			
9				13. Use trigonometric tables for values of ratios (optional).			

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MATHEMATICS: GRADES 7 and 8

AREA: GEOMETRY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	7,8		Demonstrate an understanding of the principles of geometry and measurement				
7,8		V-1 V-3	Recognize simple plane geometric figures, parallel lines, perpendicular lines, and right angles				
7,8		II-4	Find perimeters				
7,8		II-5	Find rectangular areas				
7,8		V-2	Recognize common solid geometric figures				
			Identify, Name, or Measure Angles or Figures				
7				1. Identify, name, and measure right, acute, and obtuse angles.			
8				2. Measure angles of various measure in addition to measuring right, acute, and obtuse.			
8				3. Identify adjacent, vertical, complementary, and supplementary angles.			
7				4. Identify figures and parts of figures that are congruent.			
8				5. Identify corresponding parts of congruent figures.			

MATHEMATICS: GRADES 7 and 8

AREA: GEOMETRY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
			Constructions	6. Construct right, acute, and obtuse angles. 7. Expand constructions to include, but not be limited to, construction of an angle equal to a given angle. 8. Construct the perpendicular bisector of a line segment. 9. Construct a line perpendicular to a given line at a point on the line. 10. Construct a line perpendicular to a given line from a point outside the line.			
			Polygons	11. State important properties of triangles and other polygons. 12. Determine lines of symmetry. 13. Classify triangles and other polygons according to important properties.			

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MATHEMATICS: GRADES 7 and 8

AREA: GRAPHS, MAPS, AND CHARTS

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	7,8		Demonstrate the ability to read and interpret tables and graphs and to locate ordered pairs on a grid				
		III-1	Read and interpret graphs				
		III-2	Read and interpret tables and charts				
		III-3	Interpret scale drawings				
7	7,8			1. Read and plot number pairs on a grid.			
7,8				2. Collect data and construct charts and graphs.			
7,8				3. Interpret and use data from graphs, tables, charts, diagrams, maps, and scale drawings to solve problems.			

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
	7,8		Solve problems, express problems as number sentences and identify missing or irrelevant data				
	7,8		Demonstrate the ability to read and interpret tables and graphs and to locate ordered pairs on a grid				
		III-1	Read and interpret graphs				
		I-9	Find averages of whole and decimal numbers				
7,8				1. Compute the range, mean, median, and mode of a group of numbers.			
8				2. Demonstrate knowledge of the use of statistics.			
8				3. Demonstrate knowledge of vocabulary of probability.			
7,8				4. Count or determine possible outcomes of an event.			
7,8				5. Compute the probability of an event.			
8				6. Determine the odds of an event.			
7,8				7. Demonstrate knowledge of sampling.			
7				8. Use a sample to predict an average property of a population.			

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MATHEMATICS: GRADES 7 and 8

AREA: STATISTICS AND PROBABILITY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8				9. Use a sample to determine probabilities.			
9				10. Gather data to construct a sample.			
9				11. Analyze data taken for a sample.			
9				12. Report data using a histogram.			
8				13. Read a frequency distribution chart.			
9				14. Make and read a frequency distribution chart.			

MATHEMATICS: GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

C O F F I C I E N T	S T A T I S T I C I E N T	B A S I C I E N T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S I C I E N T	S T A T I S T I C I E N T	A D V A N C E D
	7,8		Add, subtract, multiply, and divide with decimals				
	7,8		Add, subtract, multiply, and divide with fractions				
	7,8		Demonstrate the ability to compute with percents				
	7,8		Apply an understanding of geometric properties to measurement and apply an understanding of units of measure				
		I-4	Add, subtract, multiply, and divide decimals				
		I-5	Add, subtract, and multiply fractions				
		I-6	Change percents to decimals and decimals to percents				
		I-7	Determine discounts, down payments, commissions, taxes, ratios, and percents				
		I-10	Change units of measure				
		I-11	Solve problems involving time				
		II-2	Measure distance by use of a ruler				
		II-3	Measure capacities				

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MATHEMATICS: GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
		II-4	Find perimeters				
		II-5	Find rectangular areas				
		III-3	Interpret scale drawings				
		IV-1	Make approximations by rounding numbers				
		IV-2	Select reasonable answers				
		VI-1	Determine change				
		VI-2	Use comparative shopping skills				
		VI-3	Compare the cost of cash and credit purchases				
		VI-4	Allocate time and money				
		VI-5	Complete checks and money orders				
		VI-6	Solve rate, time, and distance problems				
		VI-7	Compute salaries and wages				
7,8			Time	1. Use knowledge of conversion facts to change units of time to solve problems.			

MATHEMATICS: GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
7,8				2. Add and subtract units of time and convert among the units.			
7				3. Determine beginning, ending, and elapsed time.			
7,8				4. Solve problems involving time by determining beginning, ending, and elapsed time.			
7				5. Determine elapsed time between two different time zones.			
7				6. Determine the amount of time allocated for an activity.			
8				7. Solve problems related to time allocation.			
7,8				8. Solve problems involving rate, time, distance, and miles per gallon.			
7,8				9. Measure lengths using customary and metric units.			
7,8				10. Measure capacities using customary and metric units (including bushel, peck, pint, and quart).			
7,8				11. Solve problems involving capacities.			
7,8				12. Measure weight/mass of objects in customary and metric units.			
7,8				13. Demonstrate an understanding of place value within the metric system.			

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C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
7,8				14. Make conversions among units of length, capacity, or weight/mass within the customary or metric systems.			
7				15. Add and subtract customary measures.			
8				16. Compute with customary units.			
7,8				17. Read thermometers to determine temperature in degrees Fahrenheit and degrees Celsius and solve problems involving temperature.			
7,8				18. Estimate lengths and capacities.			
7				19. Study the history of measurement.			
8				20. Relate history of measurement and applications.			
			Applications of Measurement				
7,8				21. Find perimeters of polygons.			
7,8				22. Solve problems involving perimeters.			
7,8				23. Find circumference of circles.			
7,8				24. Solve problems involving area of geometric figures.			
7,8				25. Find surface area of prisms and cylinders.			
7				26. Find the volume of prisms and cylinders.			

MATHEMATICS: GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
8			Money	27. Find volumes to include prisms, cylinders, pyramids, cones, and spheres.			
7				28. Continue work with determining, making, and counting change from practical application problems.			
8				29. Solve problems that require determination of change after a purchase.			
7,8				30. Complete information on a check and use this information in learning about checking accounts.			
9				31. Extend work on checks and money orders to include work on balancing checking and savings accounts.			
8				32. Identify and complete information on a money order.			
7,8				33. Determine amount of tax on purchases.			
7,8				34. Read a sales tax table to determine tax on a purchase.			
8				35. Figure simple income tax using tables and charts.			
7,8				36. Compute the amount of discount on a purchase.			
7,8				37. Solve problems with unit and multiple pricing to compare prices.			

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MATHEMATICS: GRADES 7 and 8

AREA: TIME, MEASUREMENT, AND MONEY

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES
7,8				38. Find amounts of commission.
7				39. Use knowledge of installment buying and down payments to compare cash and credit purchases.
8				40. Compare cash and credit purchases to include computing down payments and interest.
7				41. Compute wages based on hourly earnings and overtime.
8				42. Determine the amount of earnings based on salary, fees, and hourly wages with or without deductions.
7,8				43. Use information from budgets to solve problems.
9				44. Solve problems involving allocation of time and money.

MATHEMATICS: GRADES 7 and 8

AREA: APPLICATIONS

C OF S	S T A N	B C T	ORIGINAL SOURCES	STUDENT OUTCOMES	B A S	S T A N	A D V
			*All Stanford and BCT sources for this section have been stated in the sections where these outcomes were introduced.				
7,8				1. Solve word problems about realistic situations involving whole numbers, fractions, and/or decimals requiring the use of one or more operations.			
7,8				2. Solve word problems related to consumers to include discounts, down payments, commission, taxes, ratios, and percents.			
7,8				3. Solve word problems that involve geometry skills.			
7,8				4. Solve word problems about realistic situations requiring the use of measurement skills.			

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